New routes to topological order:

Toric code in Rydberg atoms
and
Fractional Chern insulators in moire materials.

Ashvin Vishwanath

SIMONS FOUNDATION

Ultra-Quantum Matter

Ruben Verresen
Harvard

Misha Lukin
Harvard

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Prediction of Toric Code Topological Order from Rydberg Blockade

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Department of Physics, Harvard University, Cambridge, MA 02138, USA
(Dated: November 26, 2020)

arxiv 2011.12310

PHYSICAL REVIEW RESEARCH **2**, 023237 (2020)

Fractional Chern insulator states in twisted bilayer graphene: An analytical approach

Patrick J. Ledwith, Grigory Tarnopolsky, Eslam Khalaf, and Ashvin Vishwanath
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

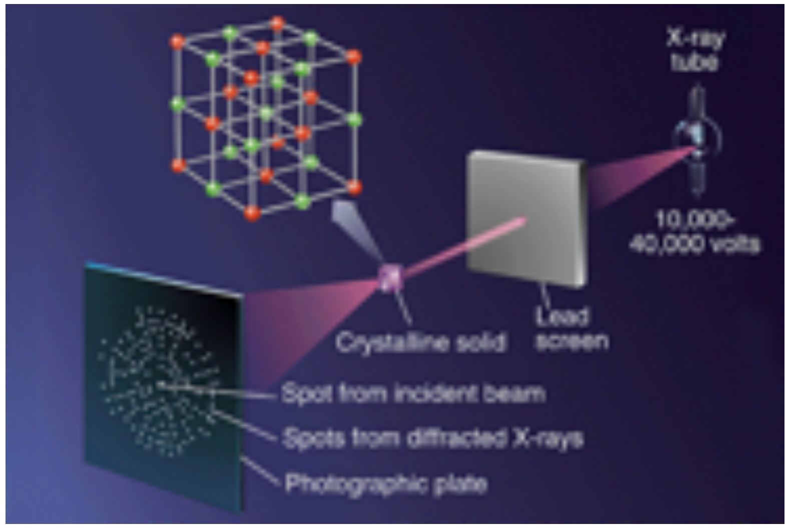
Patrick Ledwith & Grisha Tarnopolsky, Eslam Khalaf
Harvard

Symmetry Breaking States - Landau orders

Landau Order Parameter: $\langle \varphi \rangle \neq 0$

(i) Measure order parameter experimentally to diagnose a phase:

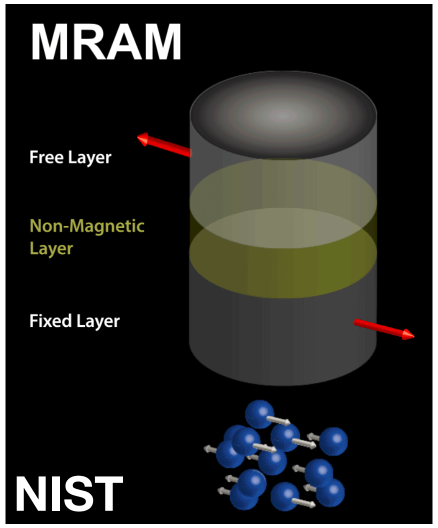
Eg. X-rays on a Crystalline solid (Density(Q))



(ii) Allows for a Classification of possible states

Eg. 230 space groups of crystalline solids

(iii) Many practical applications (eg. Magnetic memories):



1973:

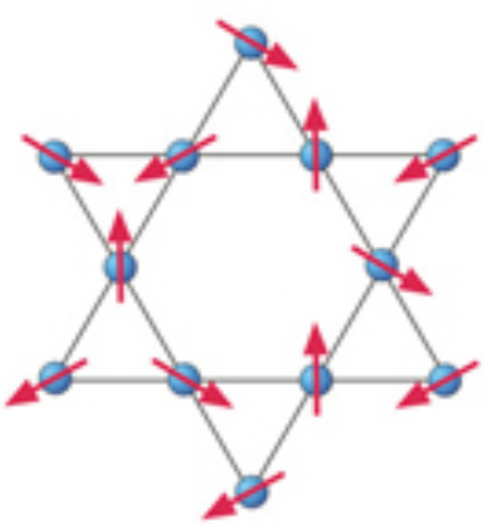
RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

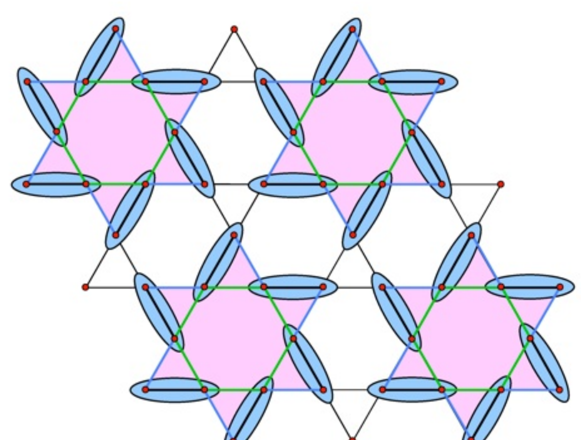
(Received December 5, 1972; Invited**)

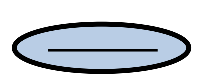
ABSTRACT

The possibility of a new kind of electronic state is pointed out, corresponding roughly to Pauling's idea of "resonating valence bonds" in metals. As observed by Pauling, a pure state of this type would be insulating; it would represent an alternative state to the Néel antiferromagnetic state for S = 1/2. An estimate of its energy is made in one case.

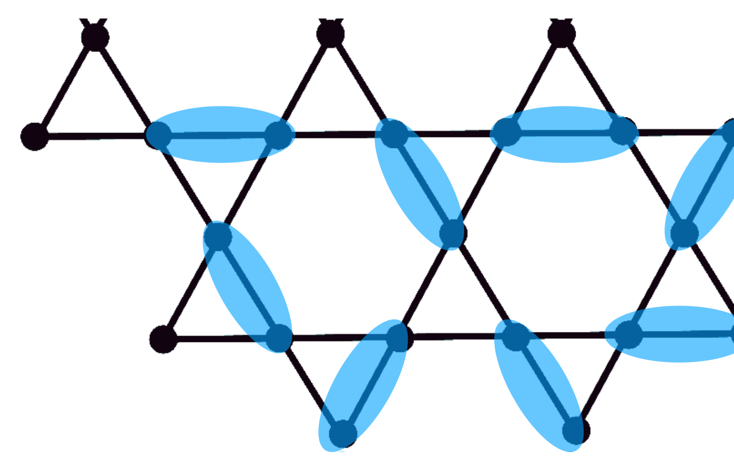


Neel order



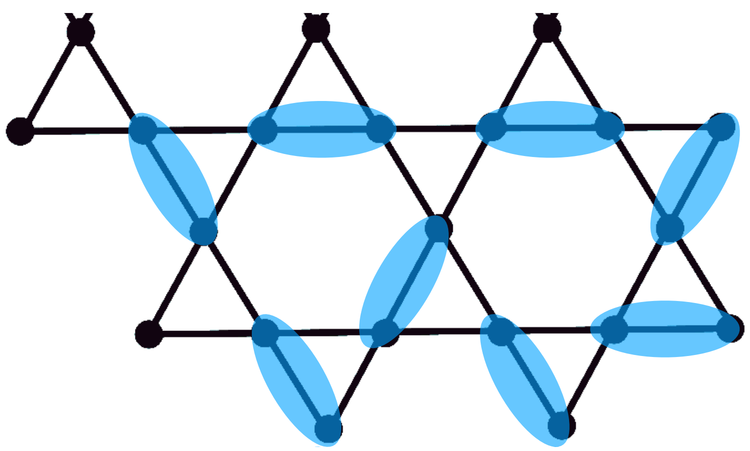
 $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle-|\downarrow\uparrow\rangle\right)$

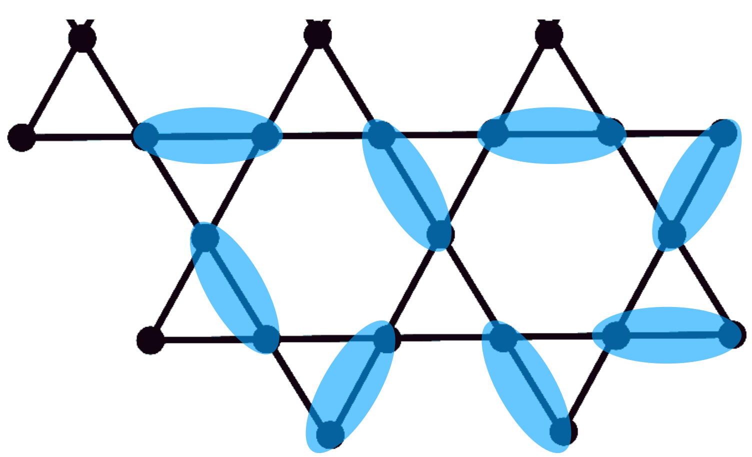
Valence Bond Crystal



Resonating Valence Bonds

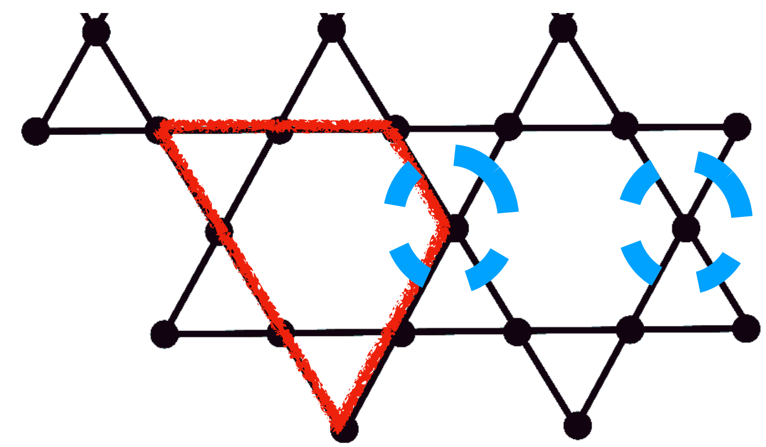
Condensates of Loops - Topological Order





Reference State

States represented by closed loops



$$\Omega = |\,\square\,\rangle + |\,{}^{\square}\,{}_{\square}\,\rangle + \dots$$

Deconfined phase of an
Emergent Gauge theory.

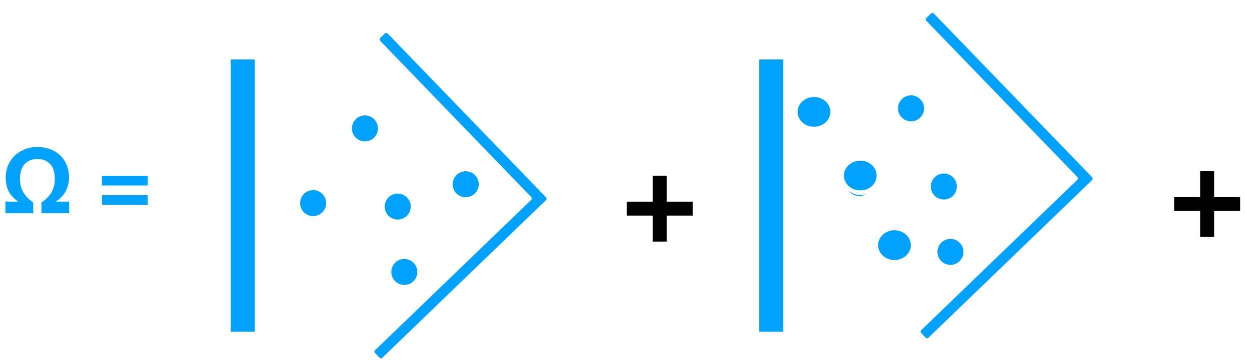
$$\nabla \cdot E = 0 \pmod 2$$

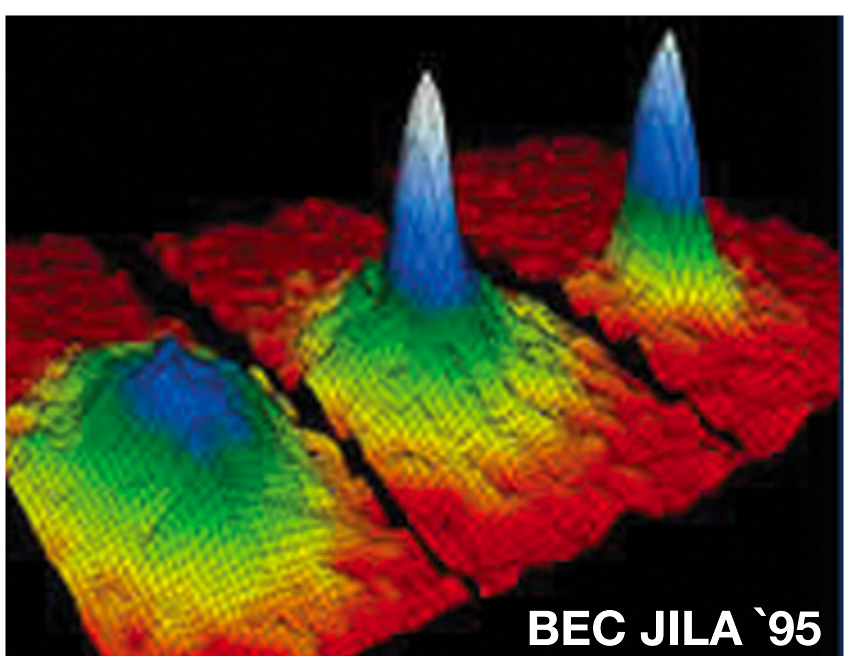
Particle Condensates - Symmetry Breaking Orders

Landau Order Parameter:

$$\langle \varphi \rangle \neq 0$$

Condensate of particles:

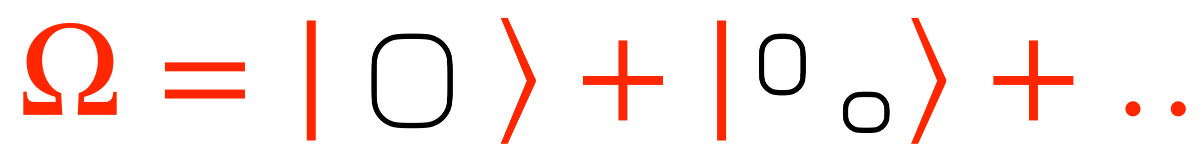


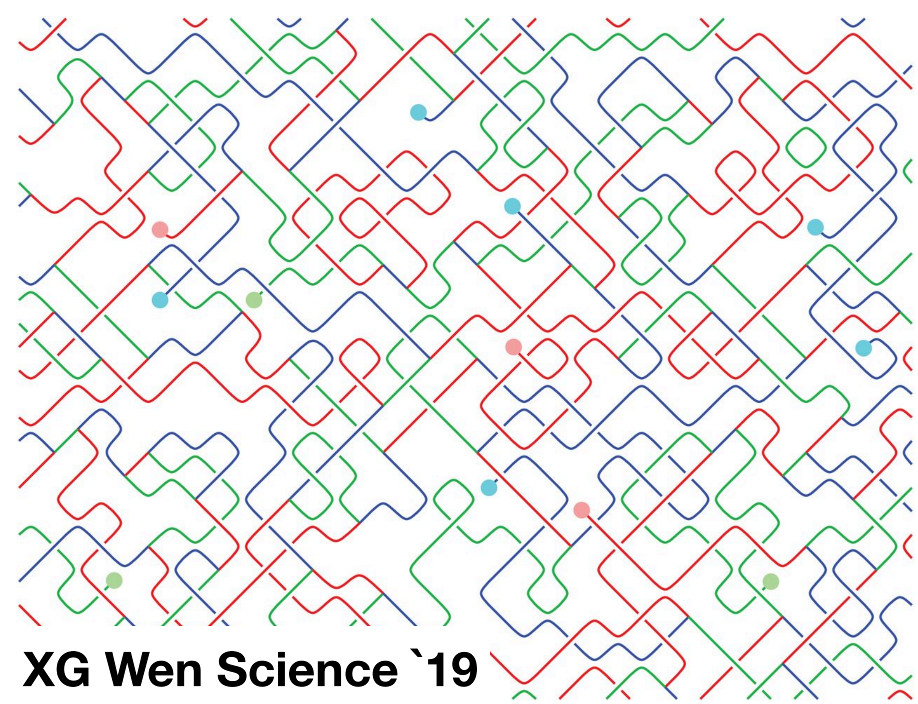


Beyond Landau Orders:

Topological Order

Condensate of closed loops:





Realization of Topological Order?

"Unfortunately, we have yet to unambiguously identify any topologically ordered phases in any real material (outside of the quantum Hall effect). This is a dire situation, and frankly is 'egg on the face' for the theoretical community."

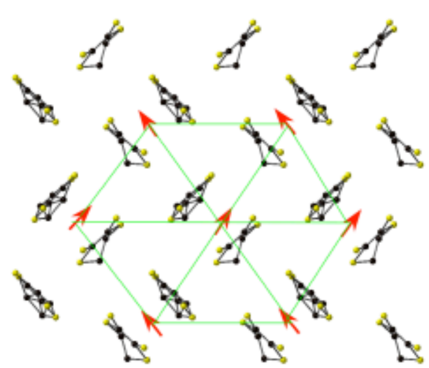
– Matthew P. A. Fisher (~2010?)

Topological Order

(Gapped) Quantum spin liquids?
Loop condensates

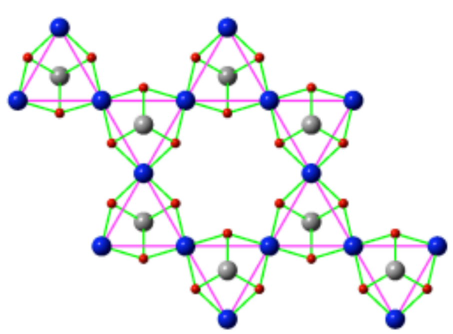
Promising candidates - but no clearcut example

(a) κ-$(ET)_2Cu_2(CN)_3$



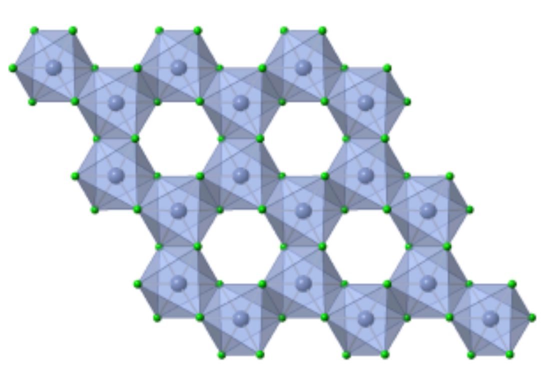
Triangular Lattice Organic

(b) $ZnCu_3(OH)_6Cl_2$



Kagome Lattice

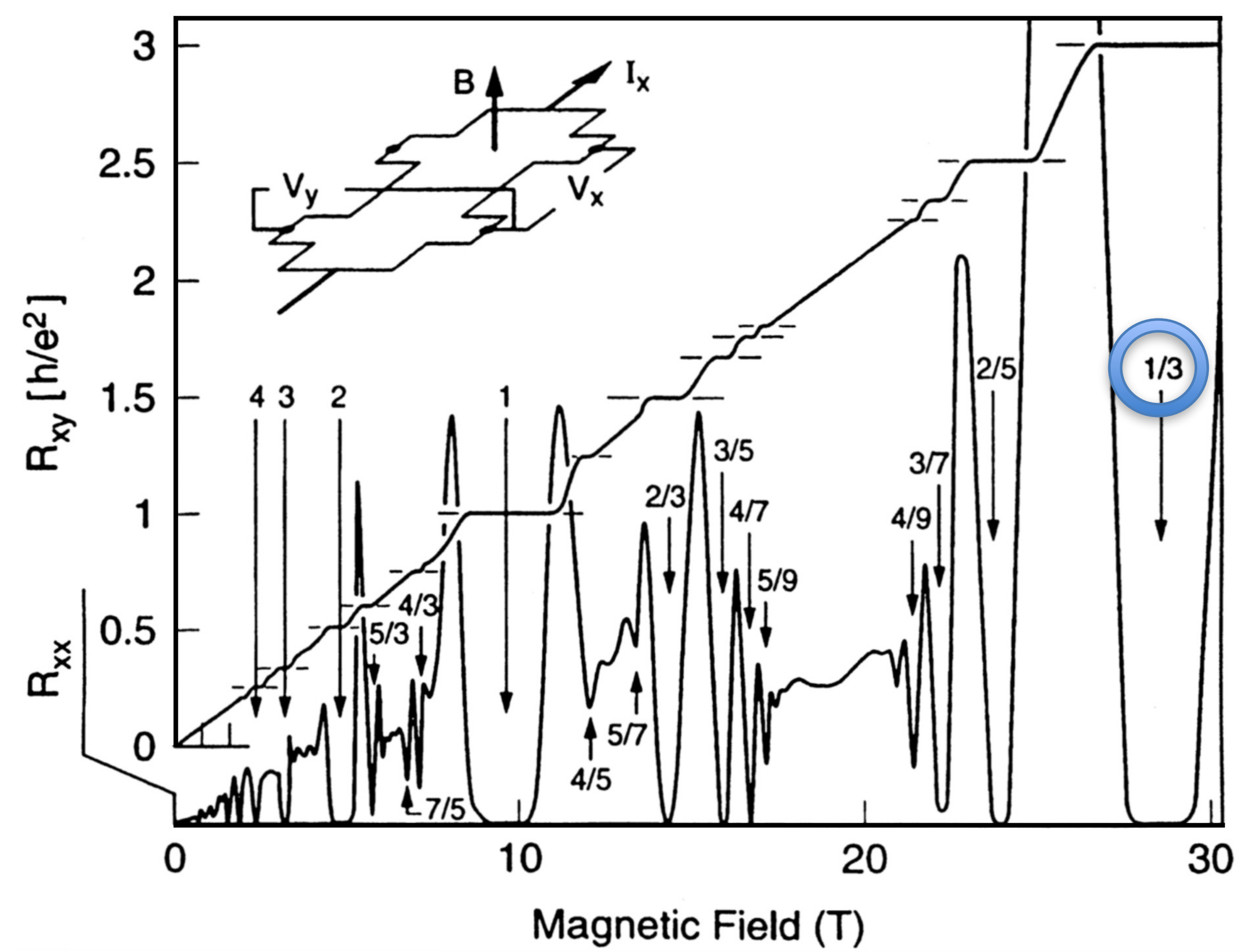
(c) α-$RuCl_3$

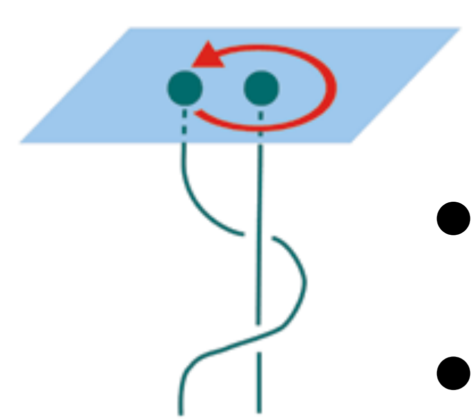


Kitaev-Honeycomb+ B field

From: Broholm et al. Science 2020

Fractional Quantum Hall





Gapped states with anyon excitations -

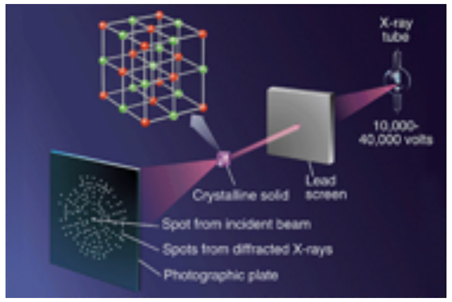
- Self statistics -unlike boson/ fermion
- *Mutual* statistics between excitations

Classical Orders versus Topological orders

Landau Order: $\langle \varphi \rangle \neq 0$

(i) Measure order parameter experimentally to diagnose a phase:

Eg. X-rays on a Crystalline solid (Density(Q))



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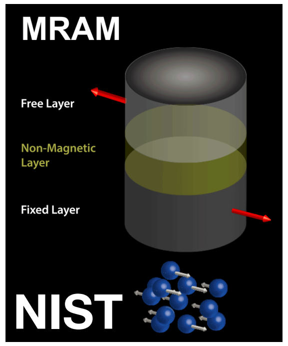
(iii) Many practical applications (eg. Magnetic memories):

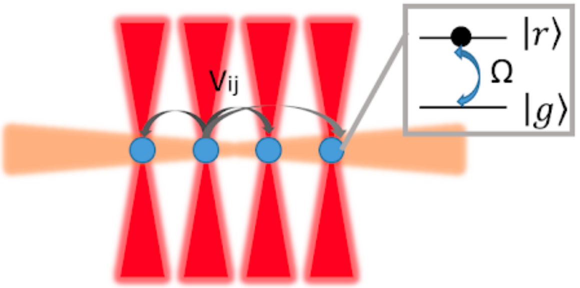


Topological Order:

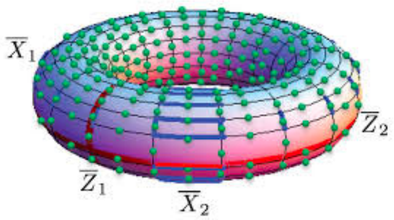
- How to measure?

(beyond Quantum Hall - edge states and quantized conductance)

- New Platforms for realization?

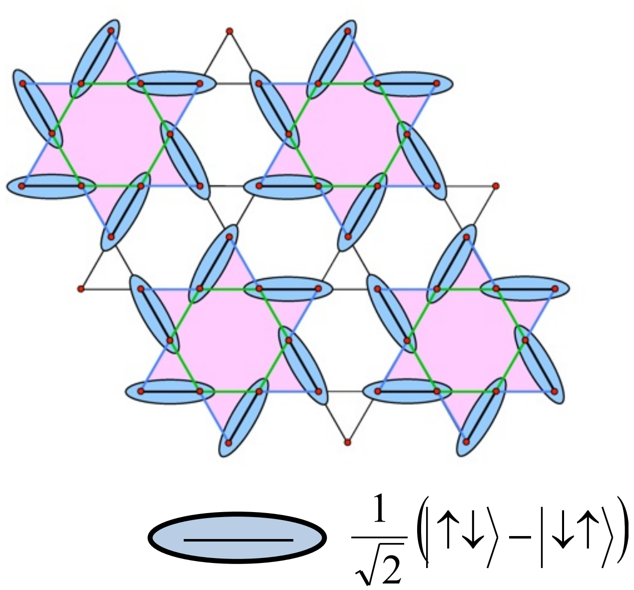


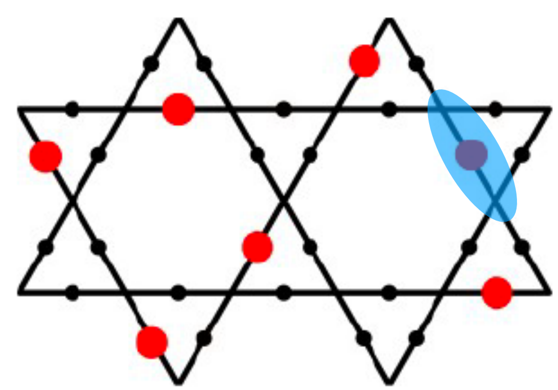
- quantum memories/computing



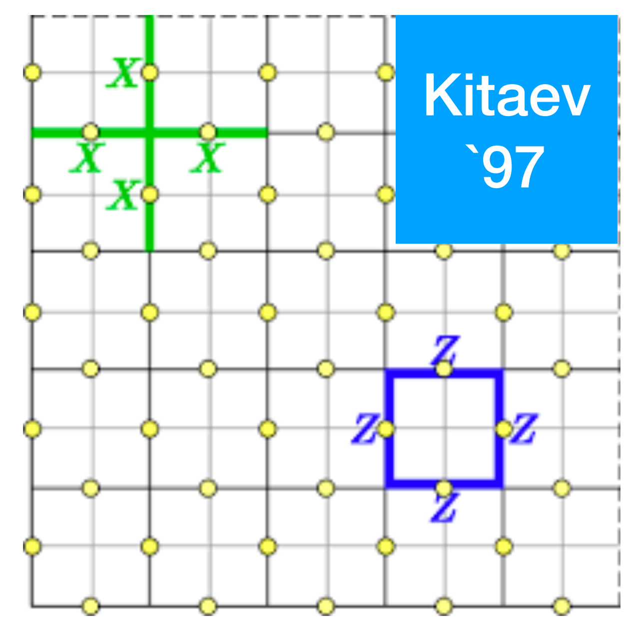
Explicit Realization of Gauge Fields

- Seek NEW platforms and models where gauge fields are *explicit* microscopic degrees of freedom eg. dimer model/ toric code





Allowed state



Toric code -
With 4 body interactions

Wegner `71; Fradkin&Shenker `79

Rokhsar and Kivelson `88; Read&Sachdev `90; Sachdev `92; Wen `91

Moessner and Sondhi 2000; Misguich, Serban and Pasquier 2001

Balents Fisher Girvin `01; Motrunich Senthil `02

Realization in Rydberg atom Array?

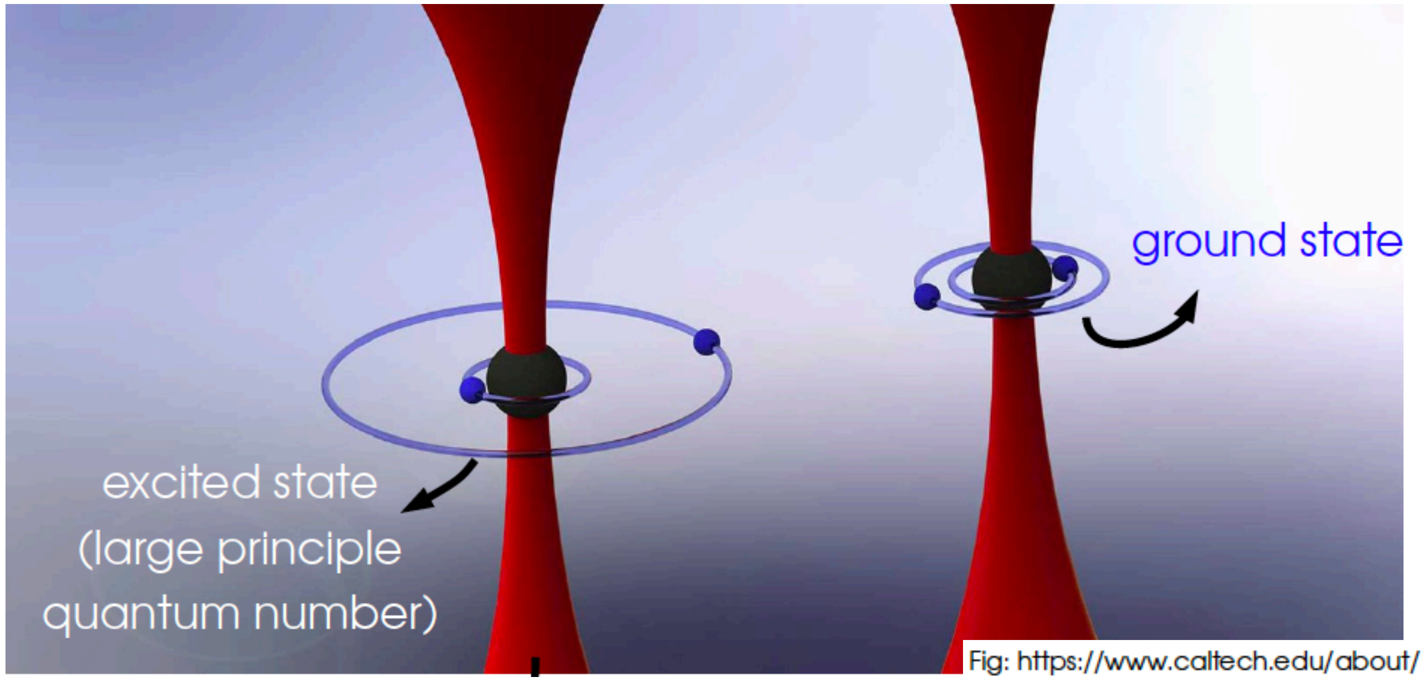
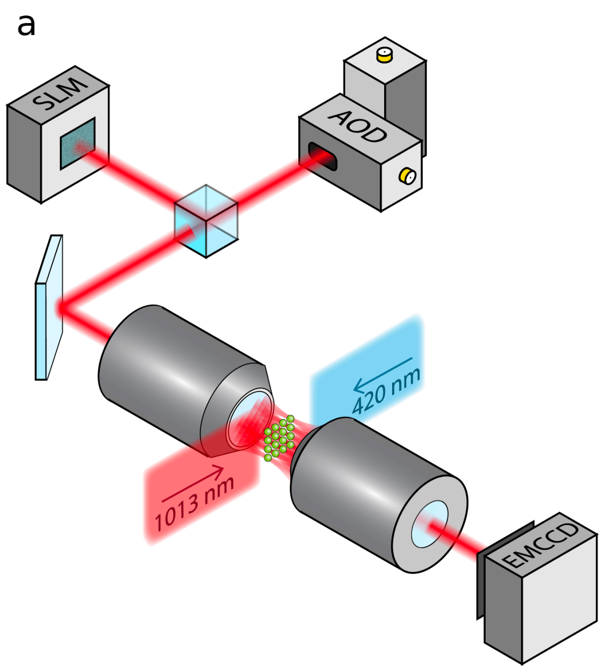
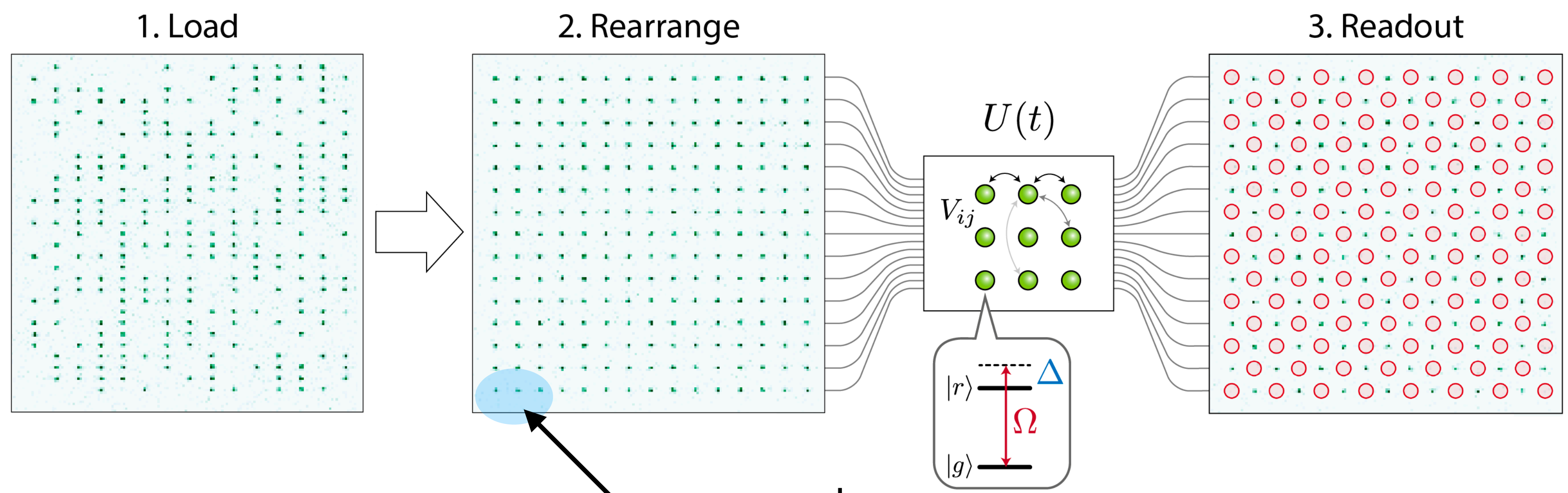
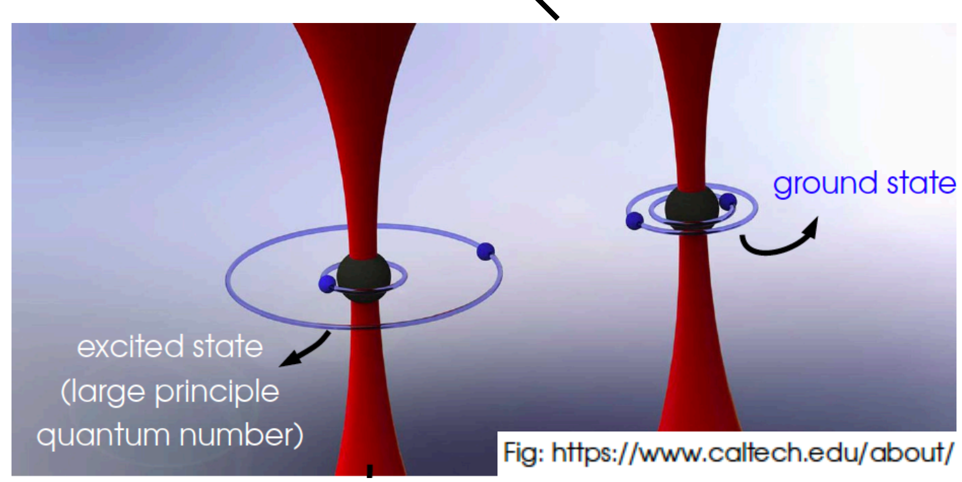


Fig: https://www.caltech.edu/about/

Rydberg atom array





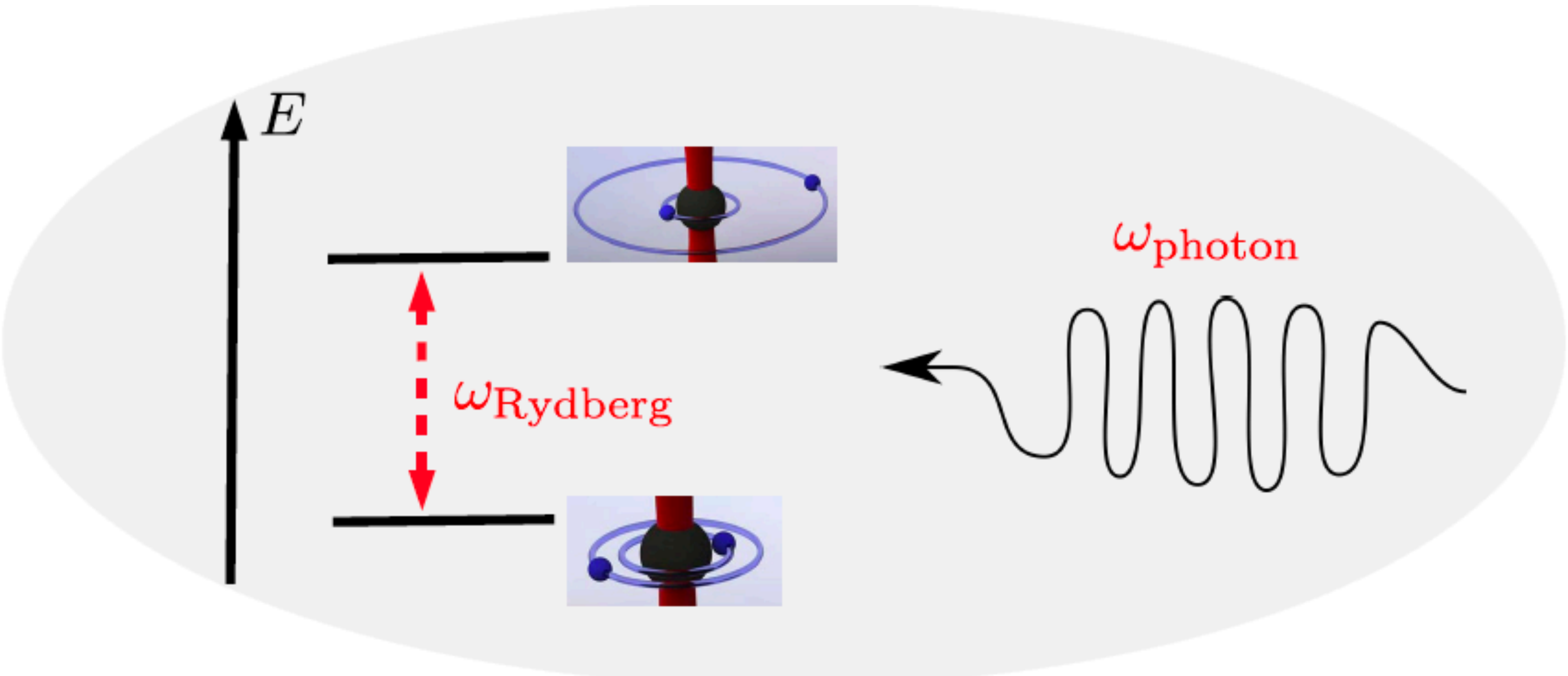


optical tweezer (traps atom)

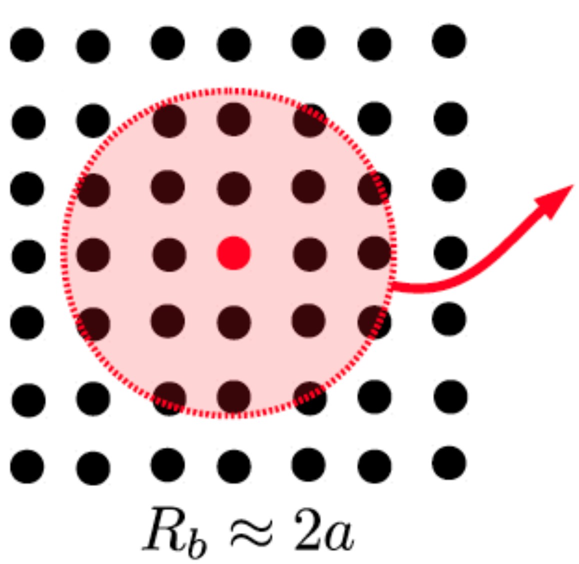
Quantum number n~70 of Rb atom - large van-der-Waals interactions

Ebadi et al. arXiv:2012.12281
Scholl et al. arXiv:2012.12268
Samajdar et al. PRL `20
Samajdar et al. PNAS `21
Ruben Verresen, Lukin, AV arxiv 2011.12310

Rydberg Hamiltonian



Described by $H = \frac{\Omega}{2}\sigma^x - \frac{\delta}{2}\sigma^z$ with $\begin{cases} \Omega = \text{Rabi frequency} \\ \delta = \omega_{\text{photon}} - \omega_{\text{Rydberg}} \\ \quad = \text{"detuning"} \end{cases}$



two nearby atoms cannot both be excited into Rydberg state

= Rydberg blockade mechanism

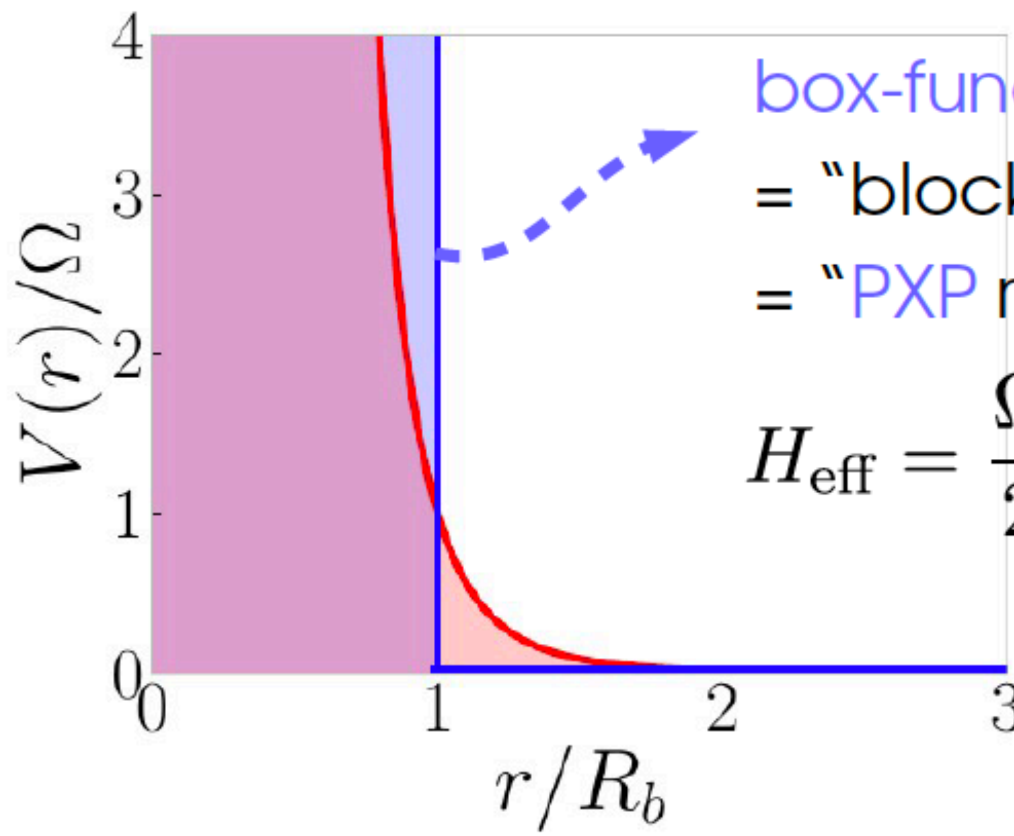
Jaksch et al., PRL (2000); Lukin et al., PRL (2001)

Rydberg Hamiltonian

$$\boxed{n_i := \frac{\sigma_i^z + 1}{2}}$$

$$H = \frac{\Omega}{2}\sum_i \sigma_i^x - \delta \sum_i n_i + \frac{1}{2}\sum_{i,j} V(|\vec{r}_i - \vec{r}_j|) n_i n_j$$

van der Waals: $V(r) \sim \left(\frac{R_b}{r}\right)^6$ $\qquad R_b = \text{blockade radius}$



box-function approximation

= "blockade model"

= "PXP model" (see literature on 1D quantum scars)

$$H_{\text{eff}} = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$

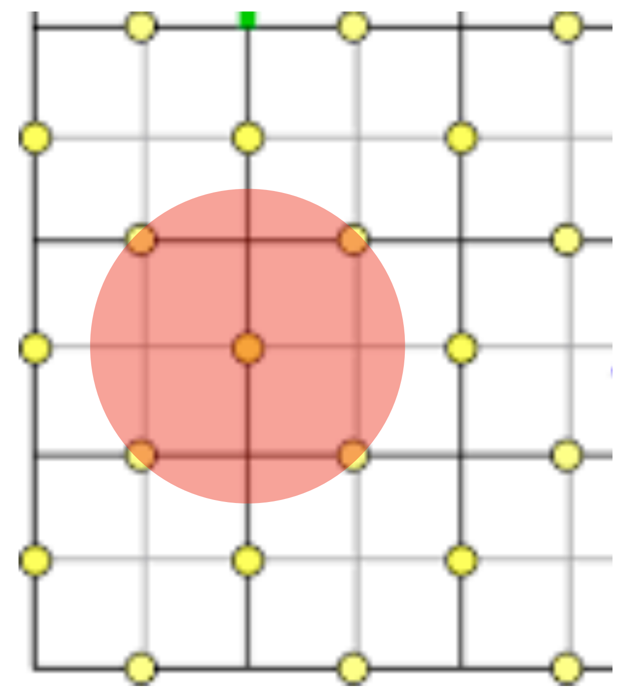
projects out double-occupation in blockade radius

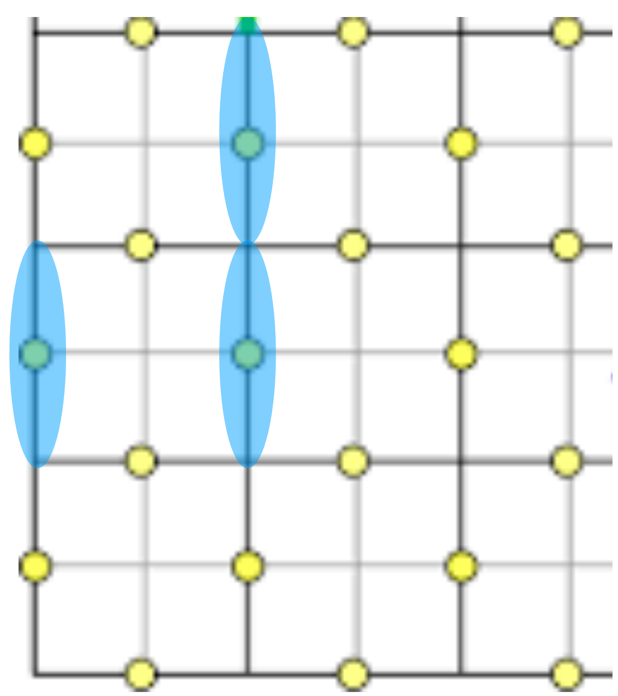
Dimer Model From Blockade?

$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta\sum_i n_i$$

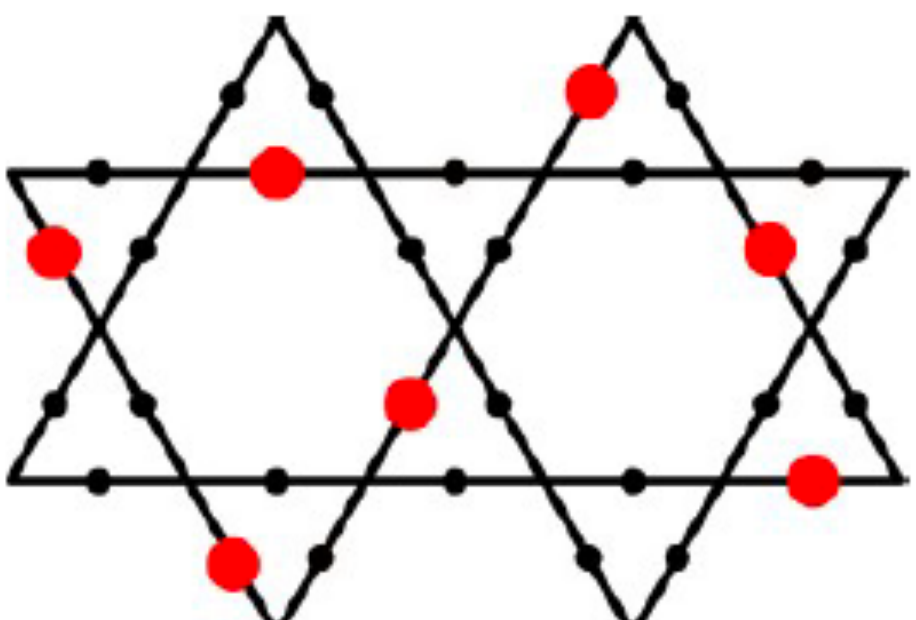
projects out double-occupation

Within radius R_b





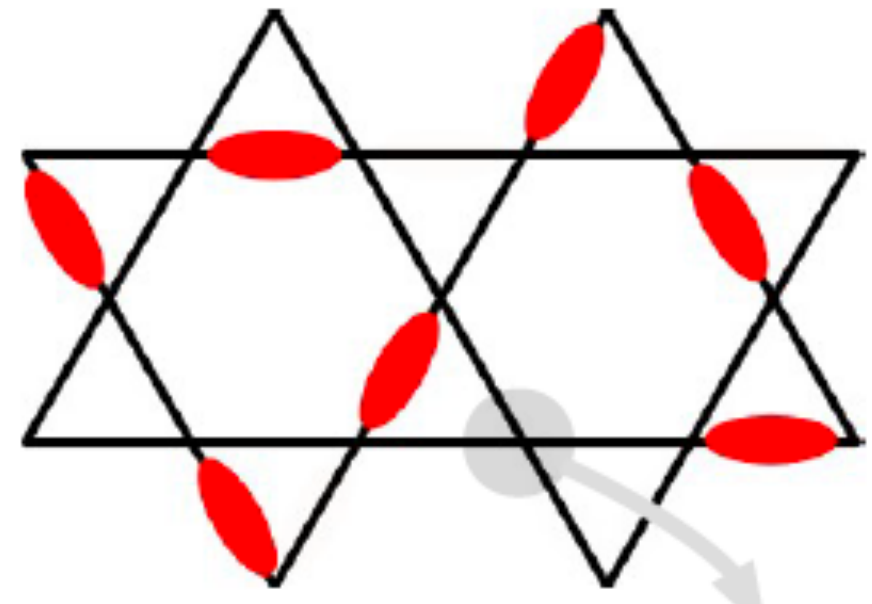
Dimer Model From Blockade



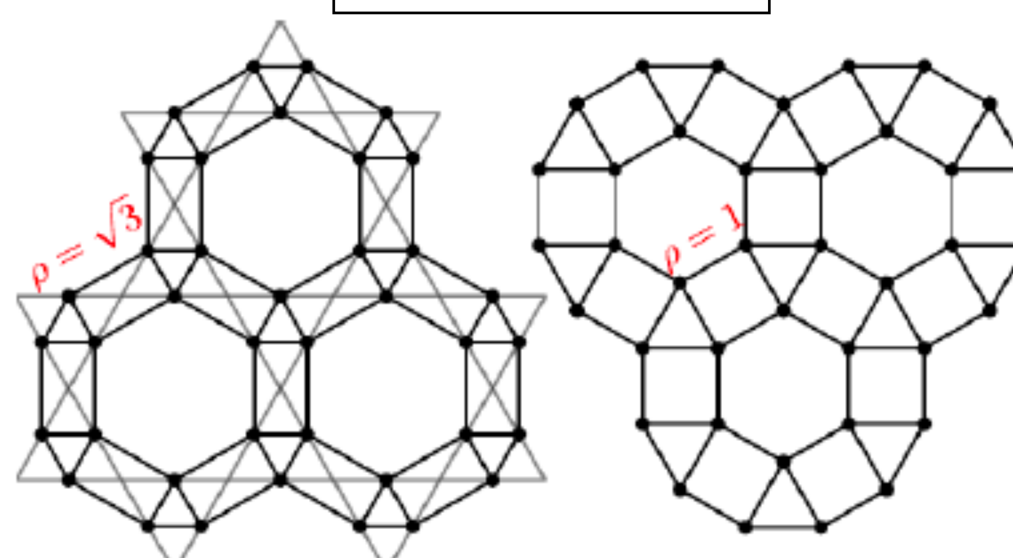
Links of the Kagome

$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$

projects out double-occupation



RUBY Lattice

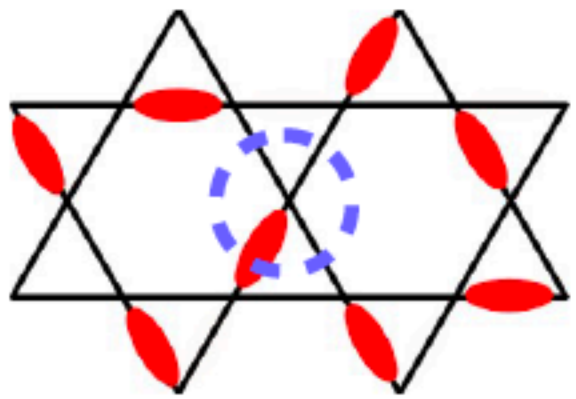


Other 2D lattices: *Samajdar et al. PRL `20. Samajdar* et al. PNAS `21

Z_2 topological order?

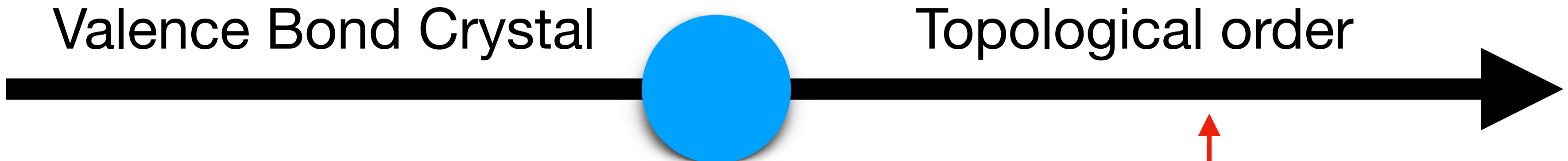
- Hilbert space - Z_2 gauge theory

The hardcore dimer constraint = Gauss law $\nabla \cdot E = 1 \mod 2$

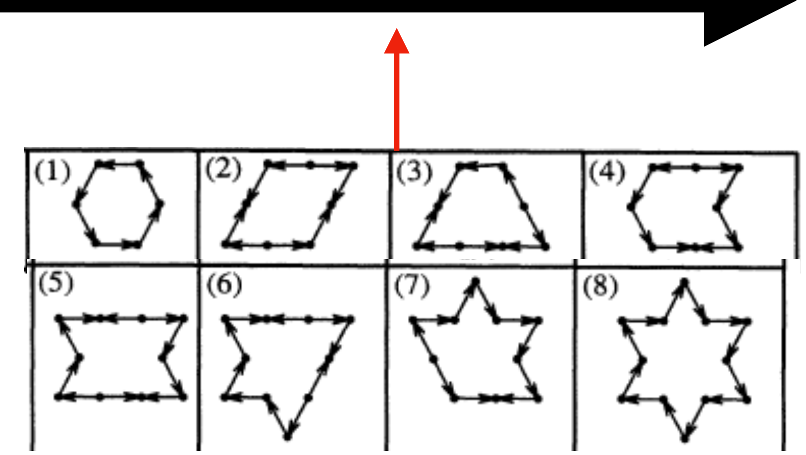


The parity around any vertex = -1

- *Dynamics* - deconfinement?



<u>Exact Solution:</u>
32 resonance terms with equal weight
Misguich, Serben, Pasquier ‘01
Zeng & Elser ‘95



Pure Dimer Model

$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$

$\delta \to +\infty$:
degenerate dimer states

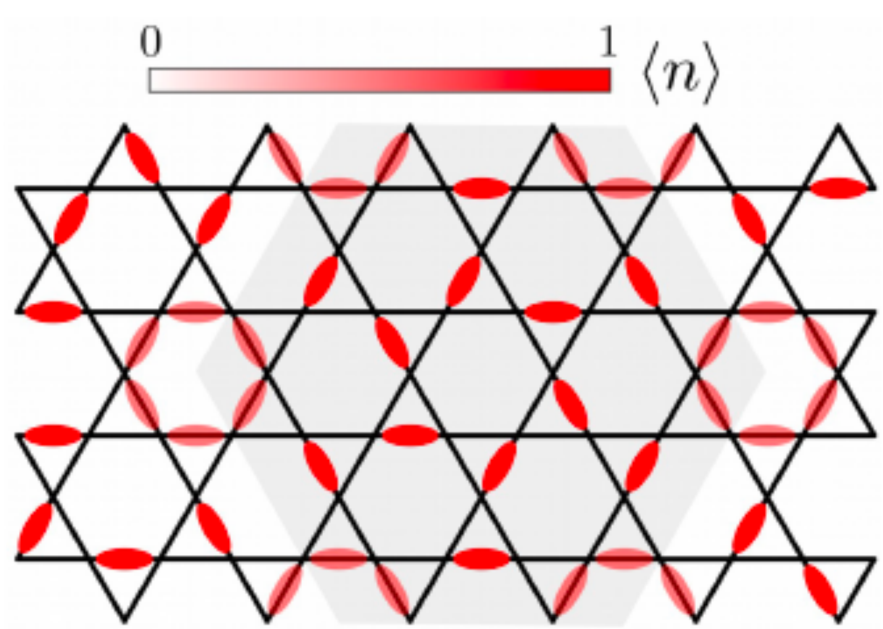
perturbation theory gives:

$$H_{\text{eff}} = -\frac{3\Omega^6}{32\delta^5}\sum_{\hexagon}\left(|\hexagon\rangle\langle\hexagon| + h.c.\right)$$

$$\boxed{\frac{\Omega}{\delta} \to 0}$$

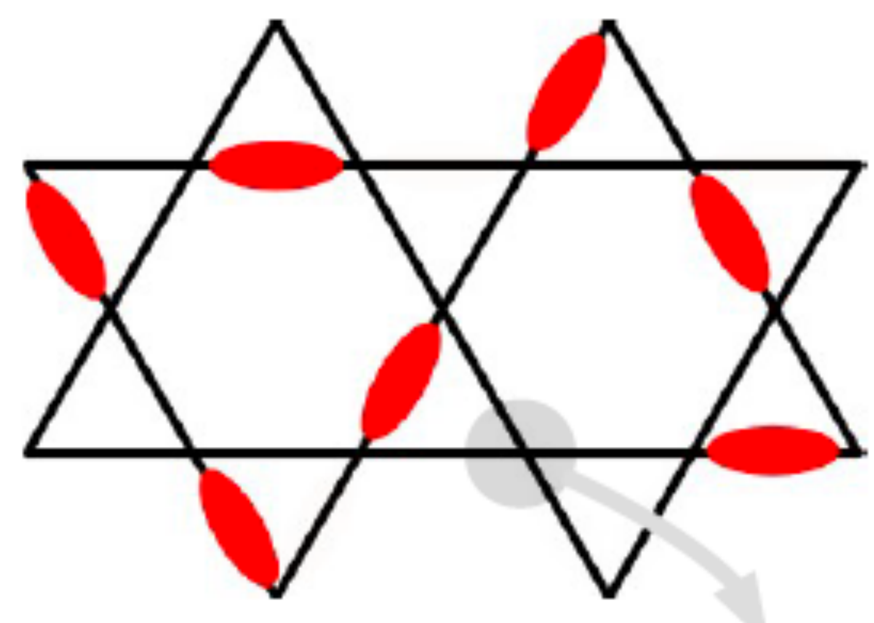
… Crystal

12 unitcells



(Nikolic and Senthil, Singh and Huse, ...)

$\boxed{\delta \downarrow}$ **Introduce vacancies - Topological order?**



Dimer state with one monomer

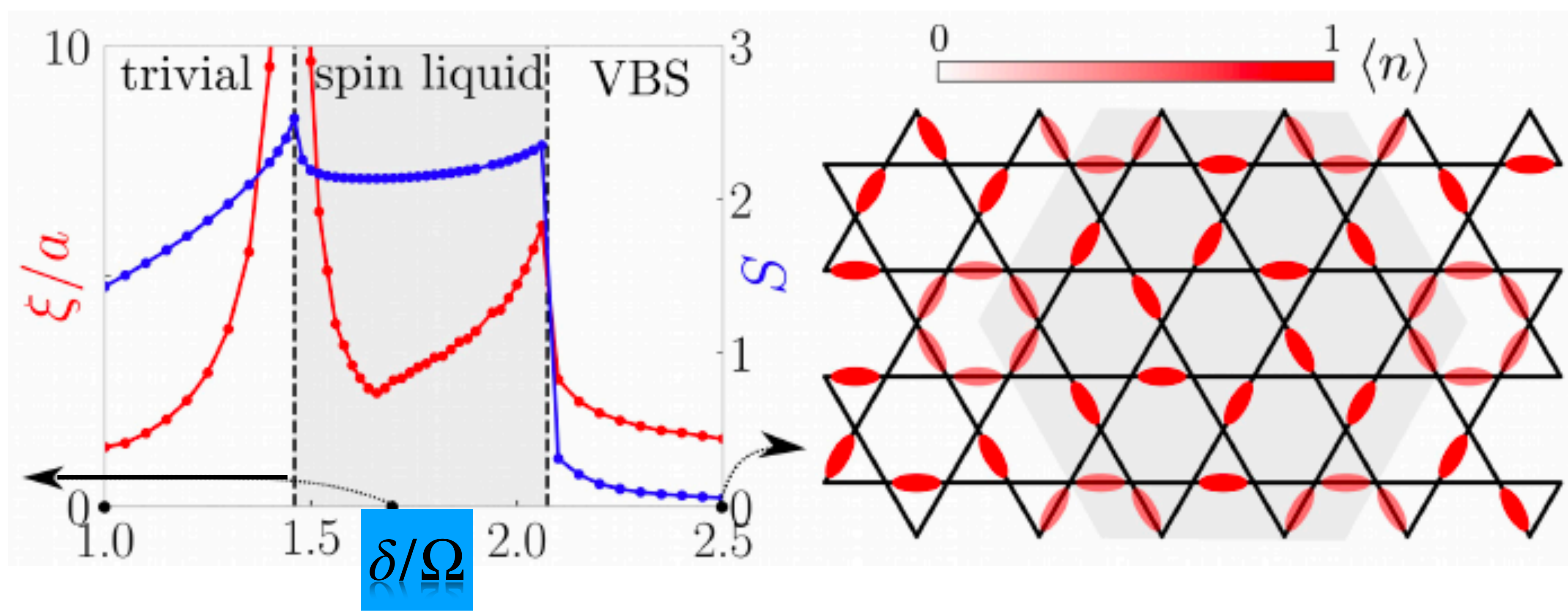
Phase Diagram of Blockade Model

$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$

we put the model on an infinitely-long cylinder

→ use density matrix renormalization group (DMRG)

(White '92, Stoudenmire '13, Hauschild '18)



intermediate featureless phase

Ruben Verresen, Lukin, AV arxiv 2011.12310

→ numerical confirmation of spin liquid

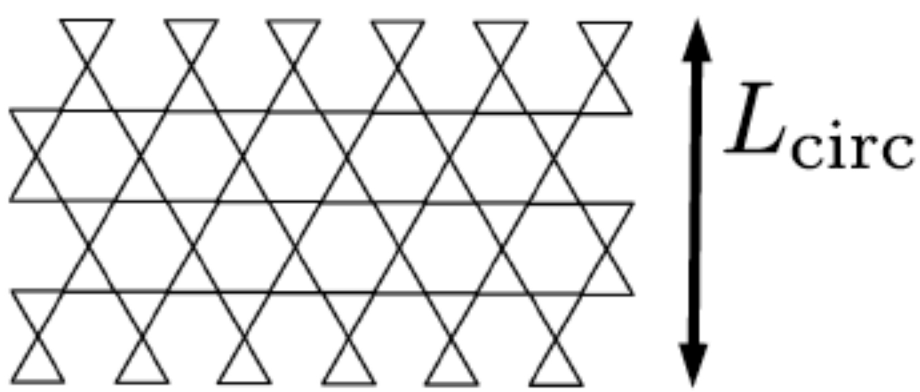
- * topological entanglement entropy
- * topological string operators
- * ground state degeneracy and modular matrices

1.Topological Entanglement Entropy

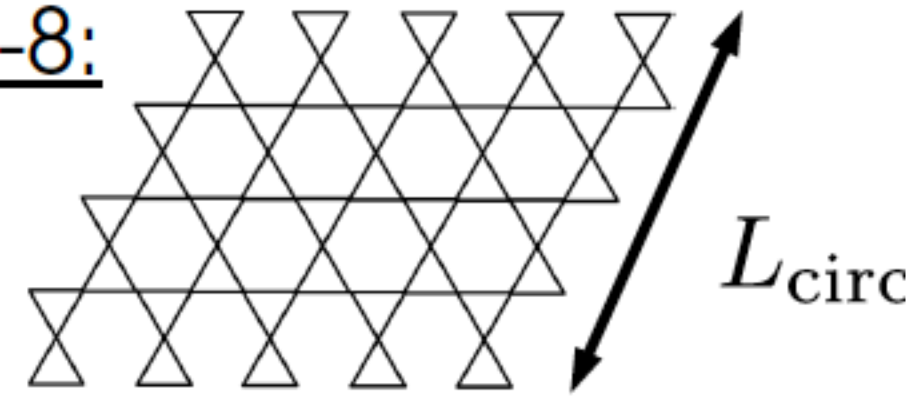
$$S_{\text{ent}} = \alpha L_{\text{circ}} - \gamma$$

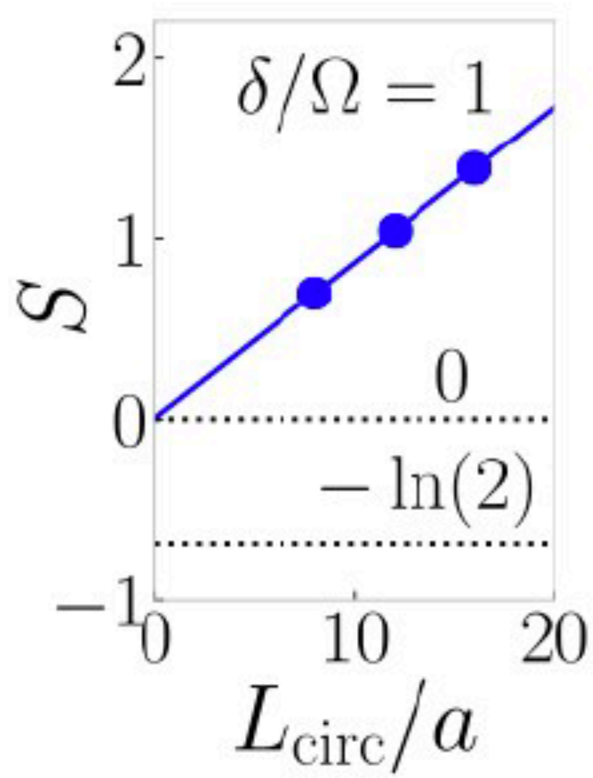
(Kitaev and Preskill, '05)

XC-8:

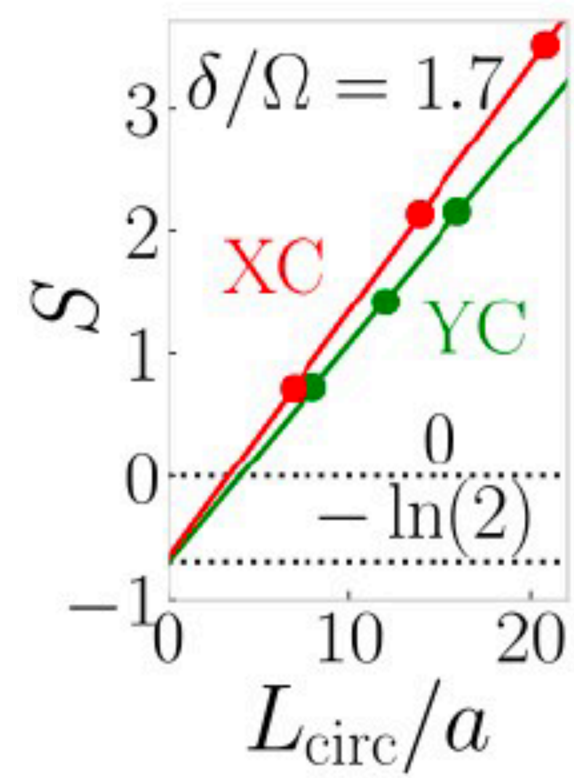


YC-8:





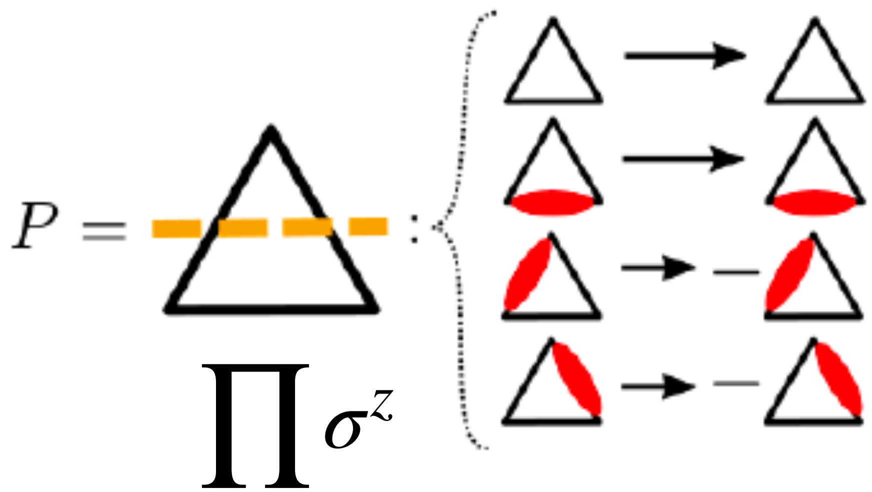
Trivial Phase

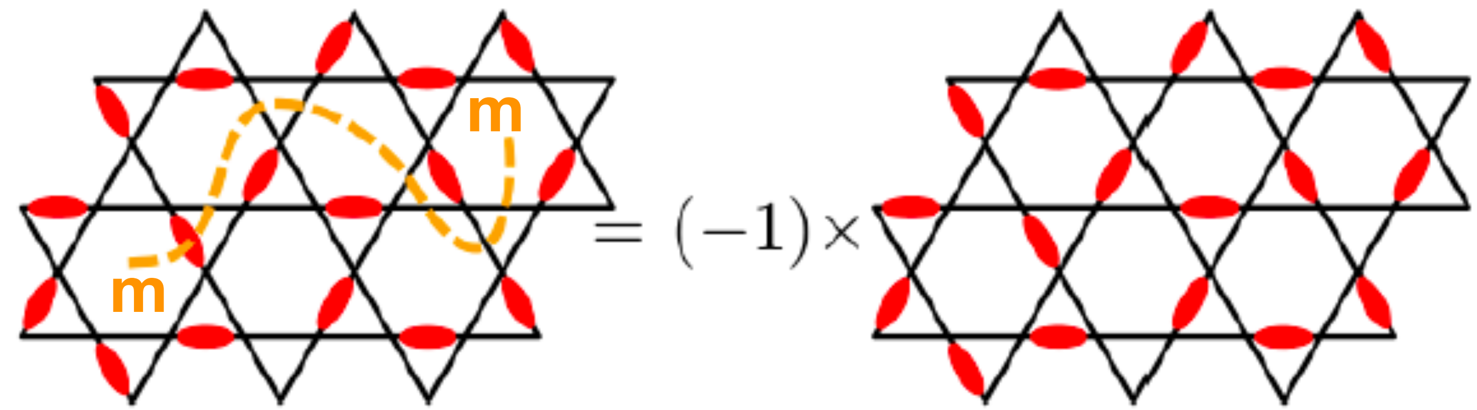


Topo. Phase

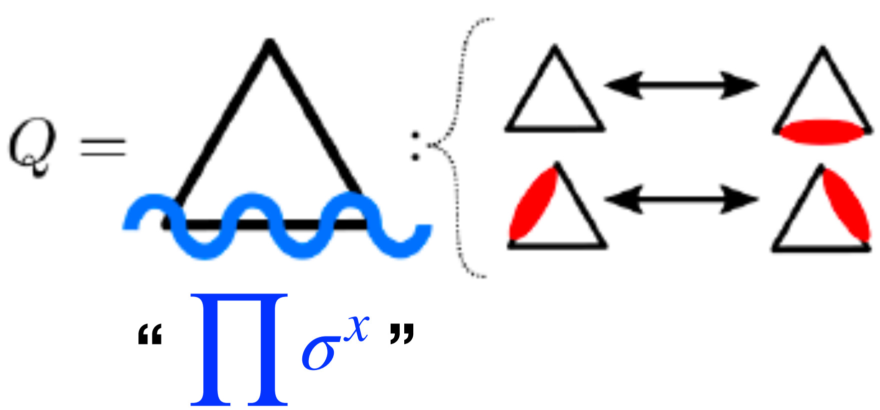
Line Operators (BFFM)

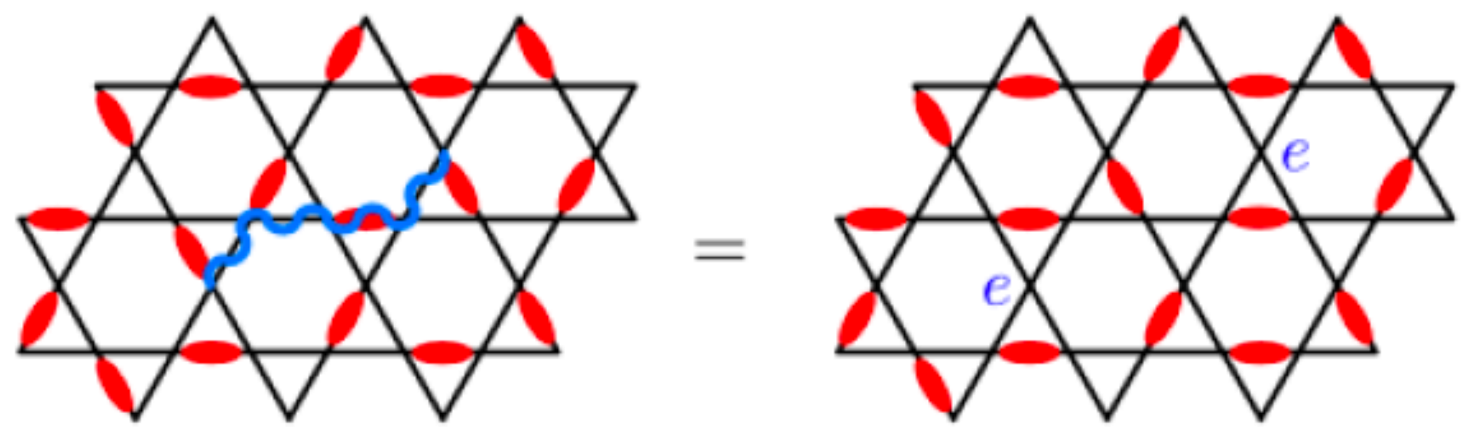
't Hooft line operator $e^{i\pi \int E}$ is diagonal in the dimer basis





Wilson line operator $e^{i \int A}$ has to anticommute → off-diagonal





Line Operators (BFFM)

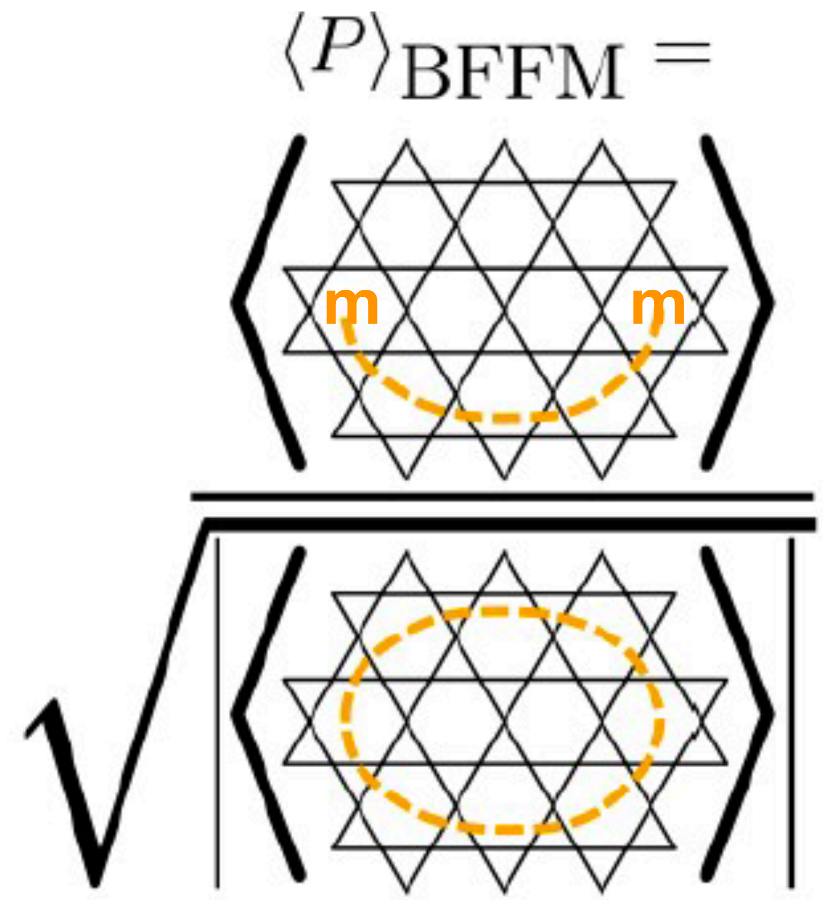
confined phase (= VBS)

= condensate of m-anyons

Higgs phase

(= trivial phase)

= condensate of e-anyons



(Bricmont and Frohlich '83; Fredenhagen and Marcu '83)

K. Gregor et al. `10

Diagnosing Phases

$\langle Q\rangle_{\mathrm{BFFM}} \neq 0$

$\downarrow$

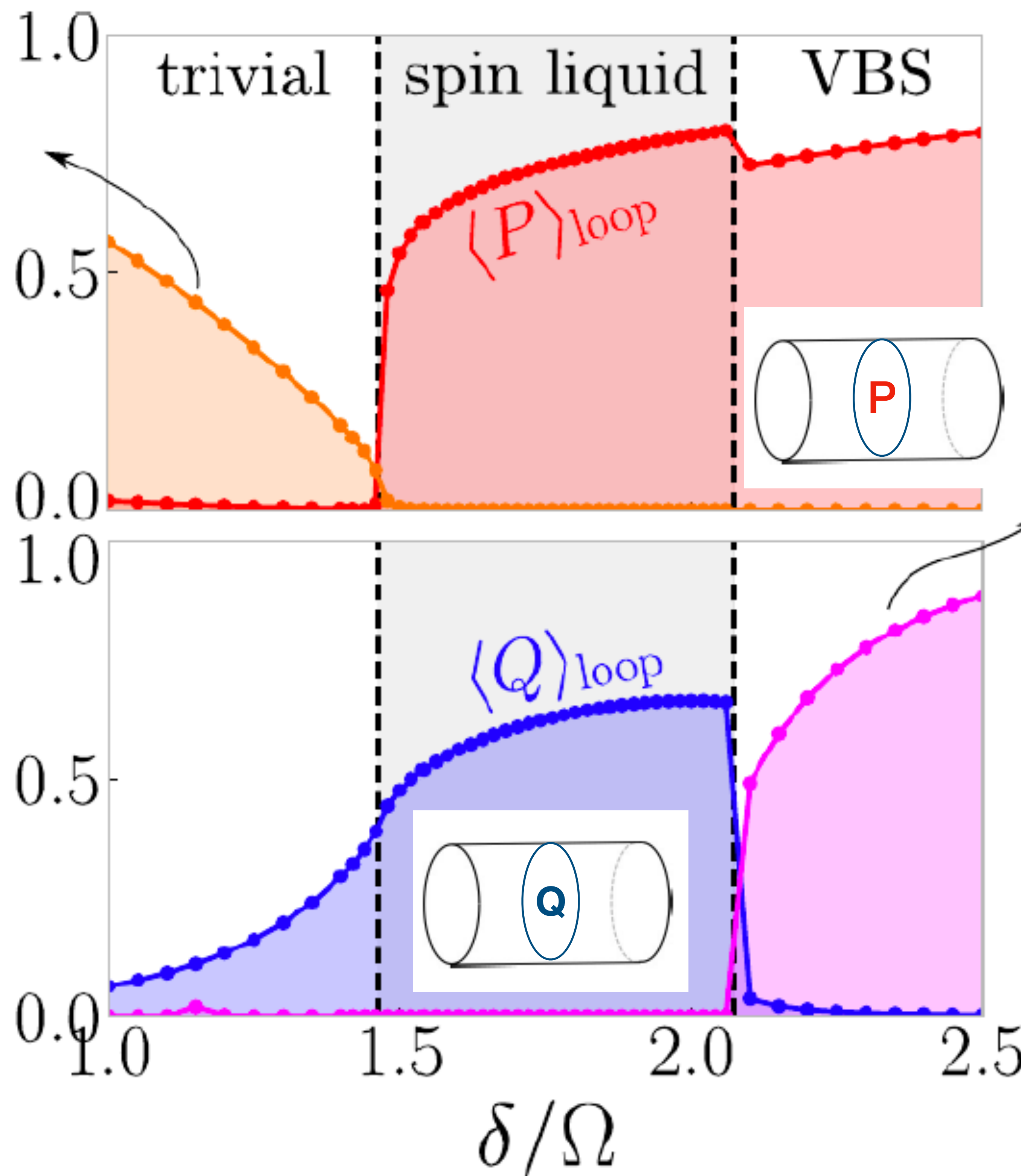
trivial phase

$=$ e-condensate

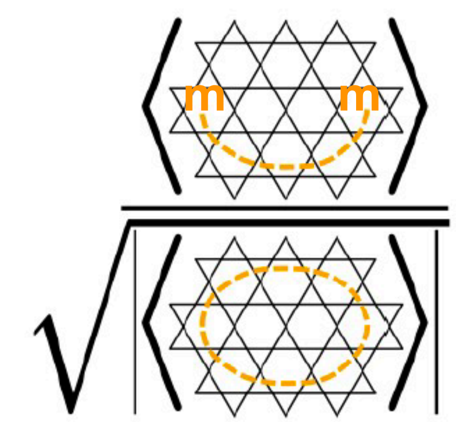
$=$ Higgs phase

$\downarrow$

suppresses $\langle P\rangle$



$\langle P\rangle_{\mathrm{BFFM}} =$



$\langle P\rangle_{\mathrm{BFFM}} \neq 0$

$\downarrow$

VBS phase

$=$ m-condensate

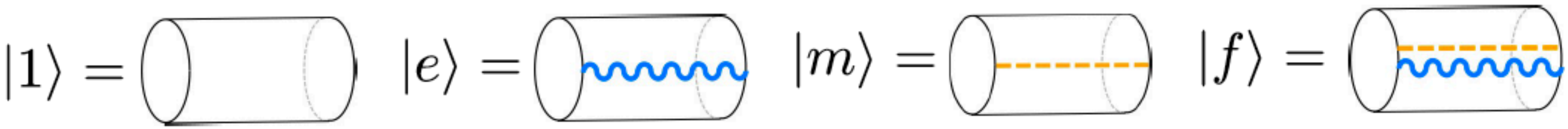
$=$ confined phase

$\downarrow$

suppresses $\langle Q\rangle$

Modular Matrices

An infinitely-long cylinder has four topological ground states



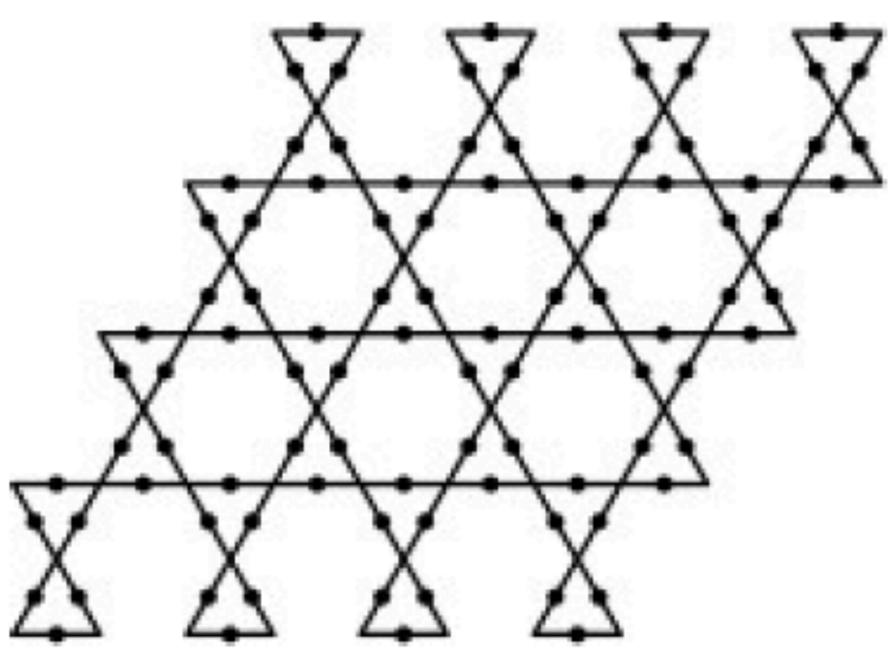
Can be diagnosed by P and Q strings around circumference

Yi Zhang et al. \`12
Cincio & Vidal \`12

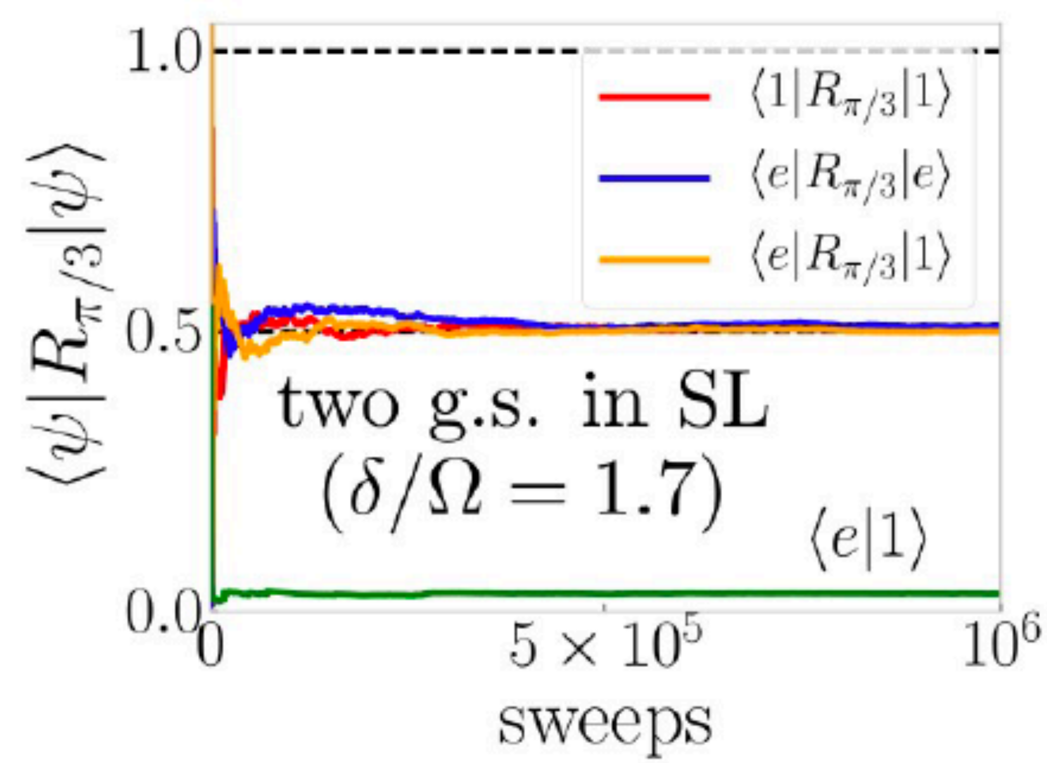
$$\begin{pmatrix} \langle 1|R_{\pi/3}|1\rangle & \langle 1|R_{\pi/3}|e\rangle \\ \langle e|R_{\pi/3}|1\rangle & \langle e|R_{\pi/3}|e\rangle \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

(XC-8)

$$\frac{m,f}{1,e} \approx 0.0001\Omega$$



(YC-8)



Towards an experimental realization

→ stability of spin liquid to van der Waals interactions

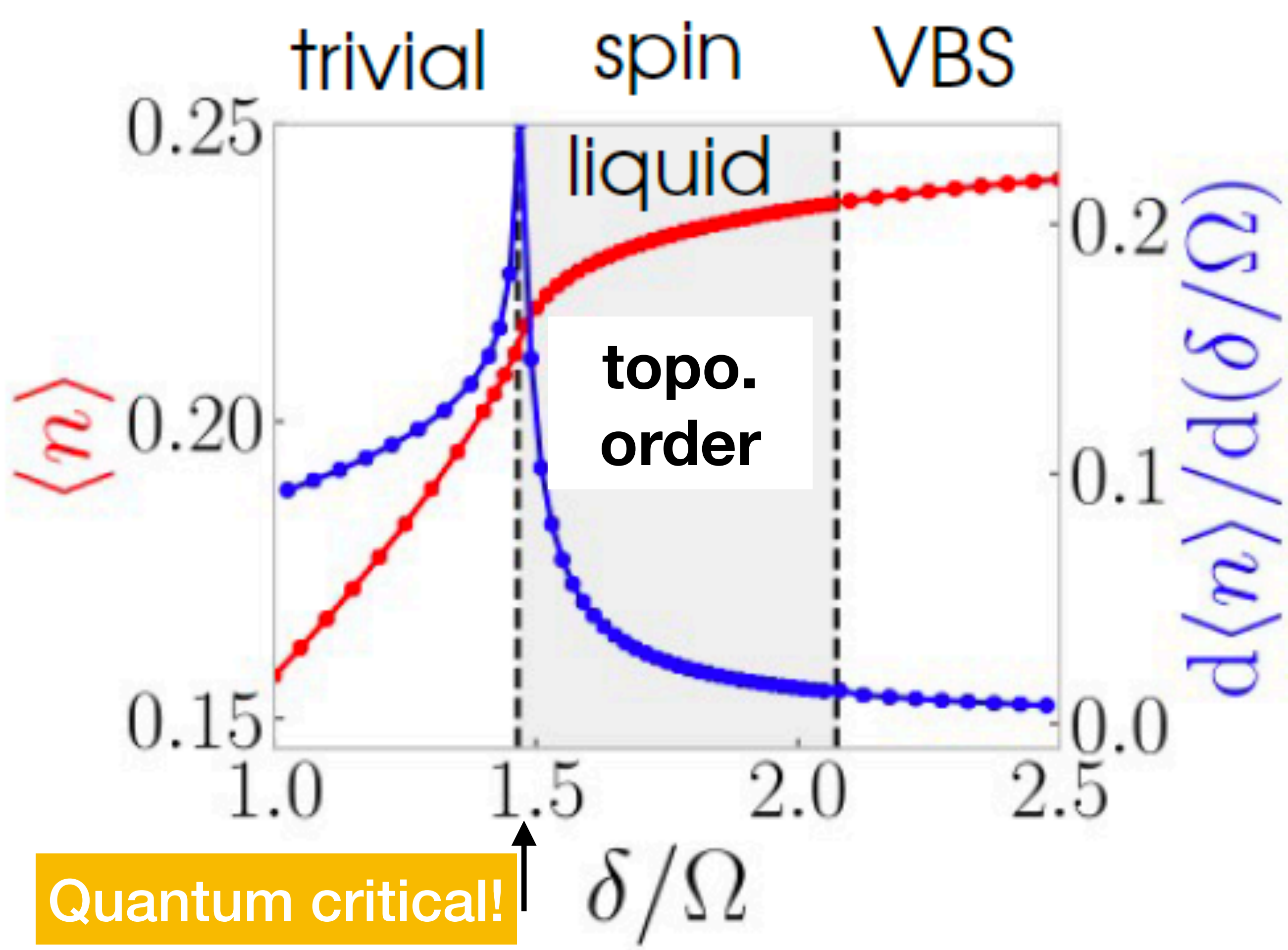
→ how to measure

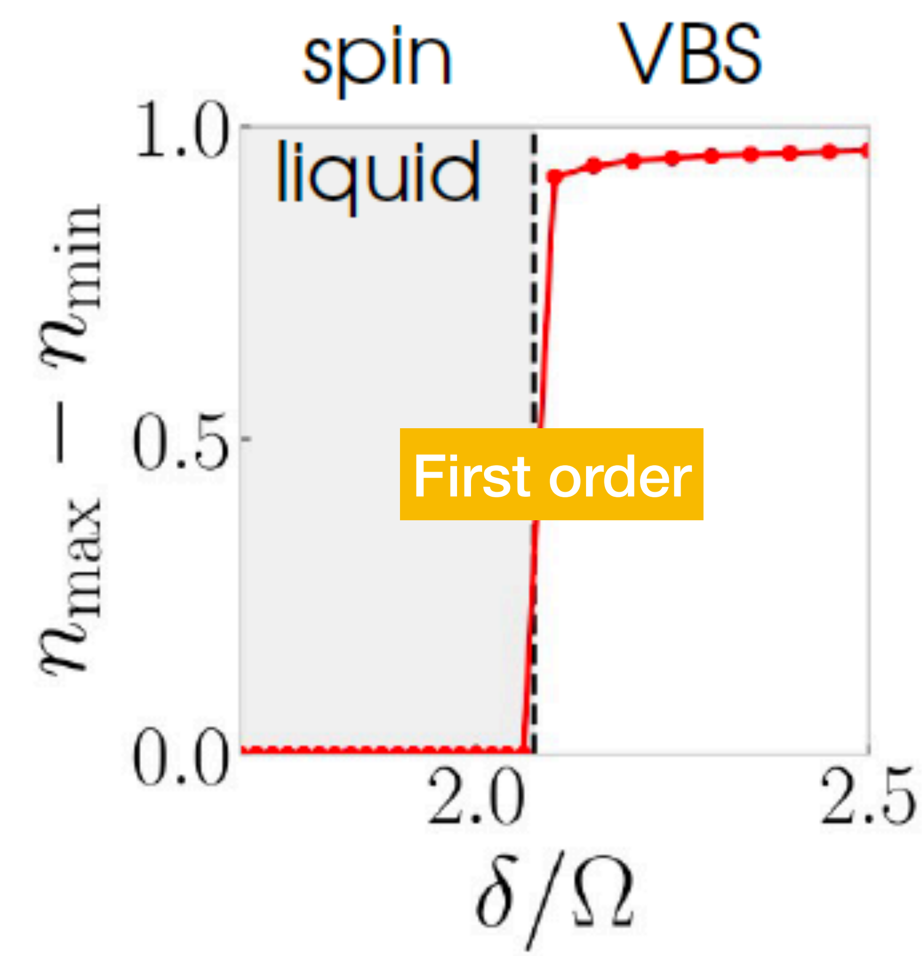
Dimer density signature

$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta\sum_i n_i$$

dimer state: $\langle n \rangle = 1/4$

with monomers: $\langle n \rangle < 1/4$





Ebadi et al. arXiv:2012.12281

Ruben Verresen, Lukin, AV arxiv 2011.12310

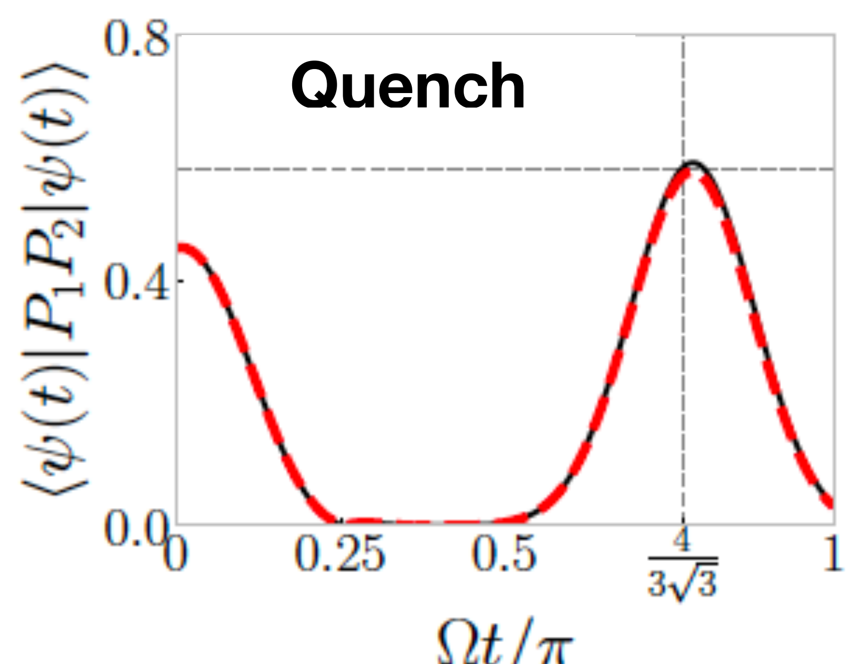
Measuring String Operators?

→ Diagonal string P measured from snapshots of atoms

→ Off-diagonal string Q can be reduced to P after rotation

$e^{iHt}\Big($  $\Big)\, e^{-iHt} =$  for $\Omega t = \frac{4\pi}{3\sqrt{3}}$

using $H = \frac{\Omega}{2} \sum_i P \sigma_i^y P$ where P only enforces blockade inside triangles!

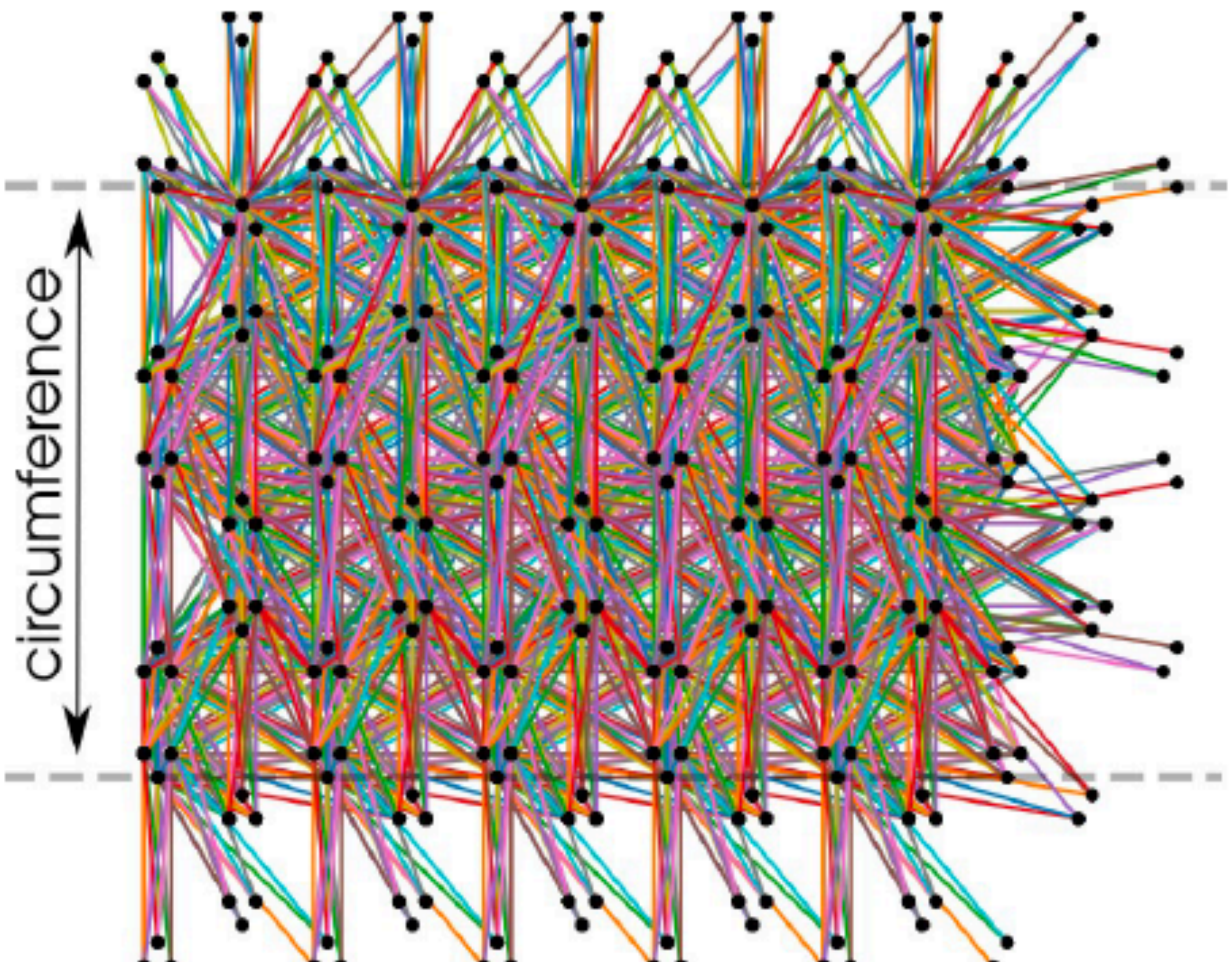


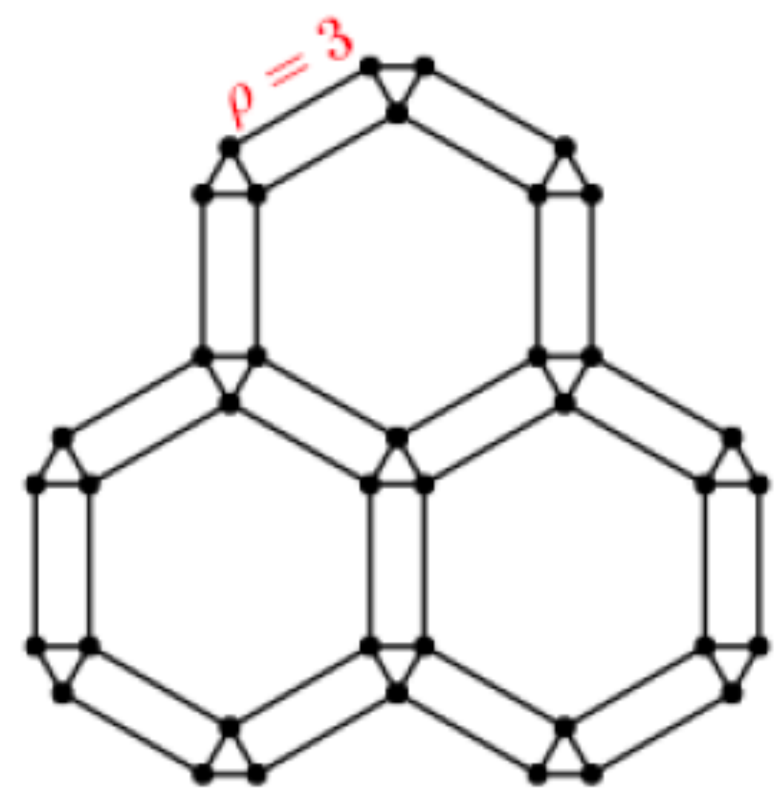
Spin liquid with van der Waals Interactions

$$H = \frac{\Omega}{2}\sum_i \sigma_i^x - \delta \sum_i n_i + \frac{1}{2}\sum_{i,j} V(|\vec{r}_i - \vec{r}_j|) n_i n_j$$

$$V(r) = \Omega \times \left(\frac{R_b}{r}\right)^6$$

Ruby lattices with $\rho > 1/\sqrt{2}$ all give same blockade model



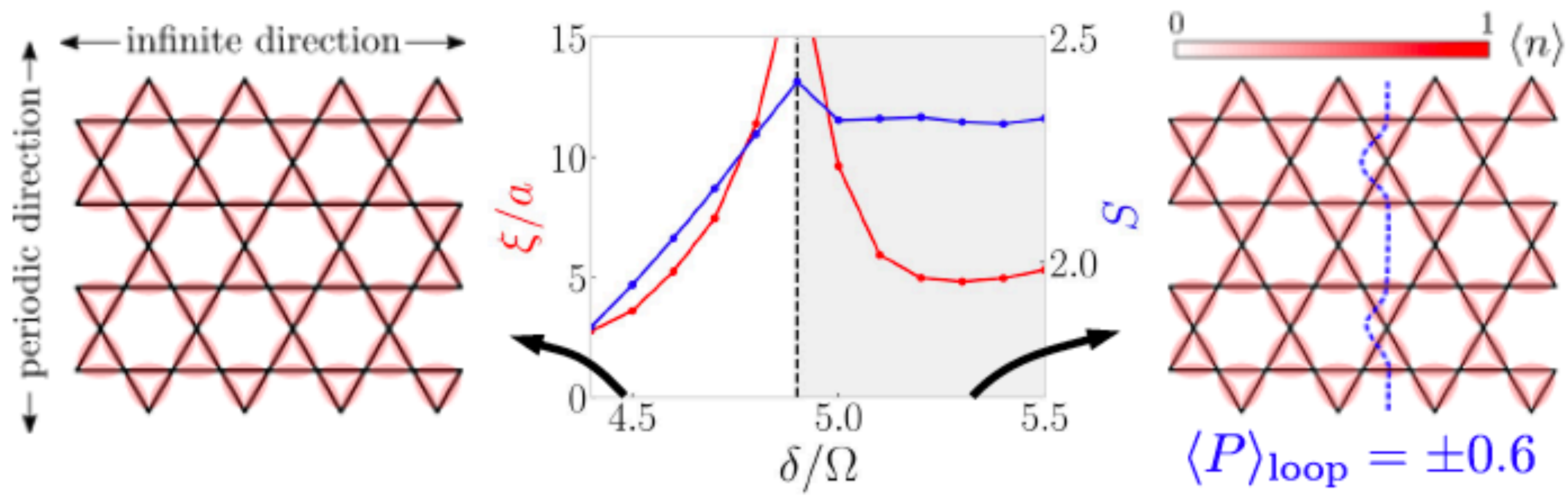


Retain: $r_1, r_2, r_3, r_4, \ldots r_{16}$

Each atom couples to 44 other sites

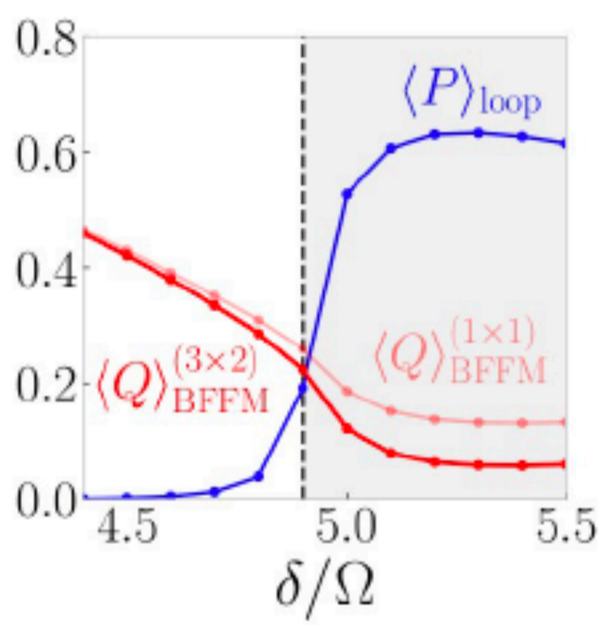
Spin liquid with van der Waals Interactions

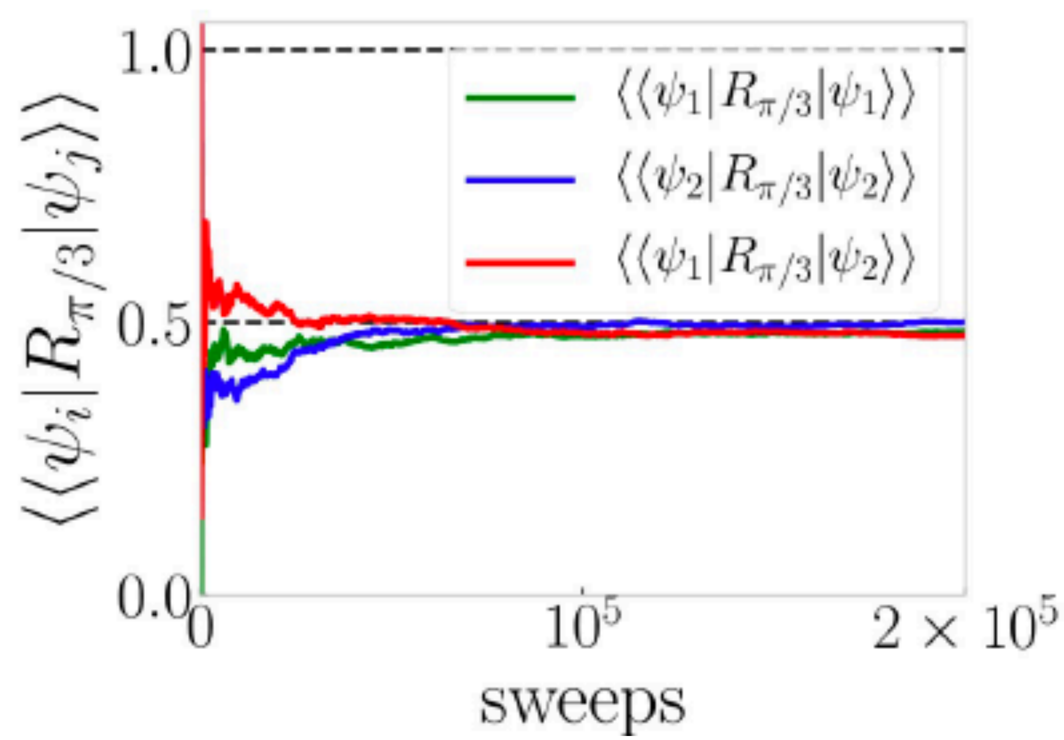
$$V(r) = \Omega (R_b/r)^6 \quad \text{with } R_b = 3.8a \text{ on ruby lattice with } \rho = 3$$



Ground States

BFFM





Modular Matrix

Ruben Verresen, Lukin, AV arxiv 2011.12310

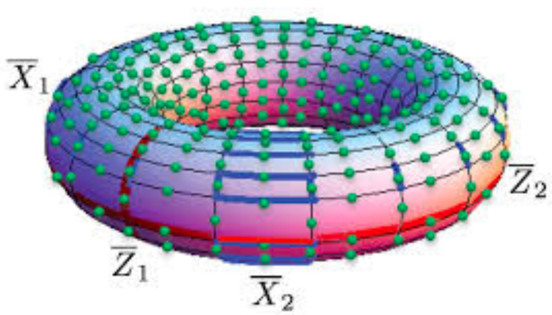
Towards a topological qubit

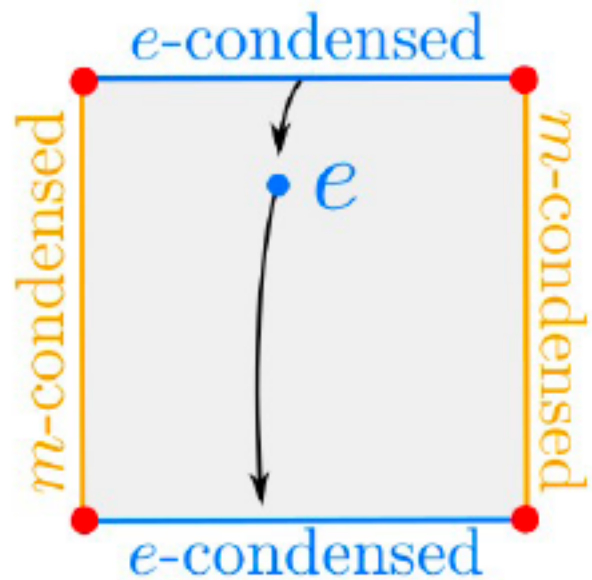
→ capturing anyons

→ topological boundary conditions

Towards Topological Quantum Bit

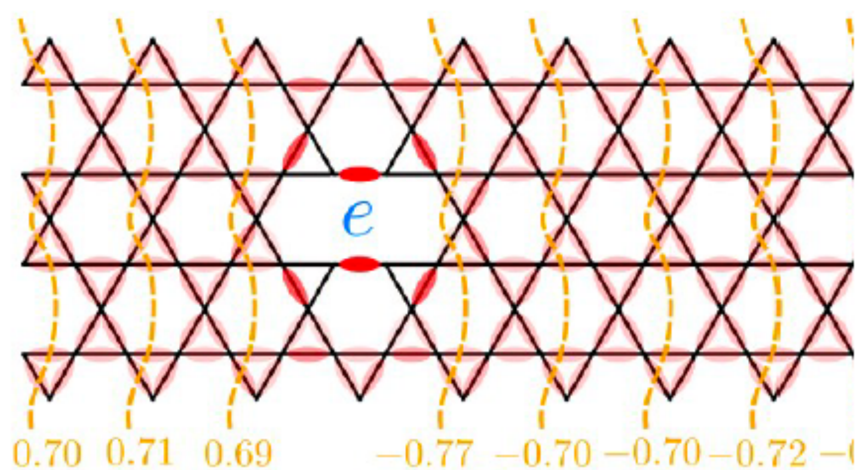
Alexei Kitaev

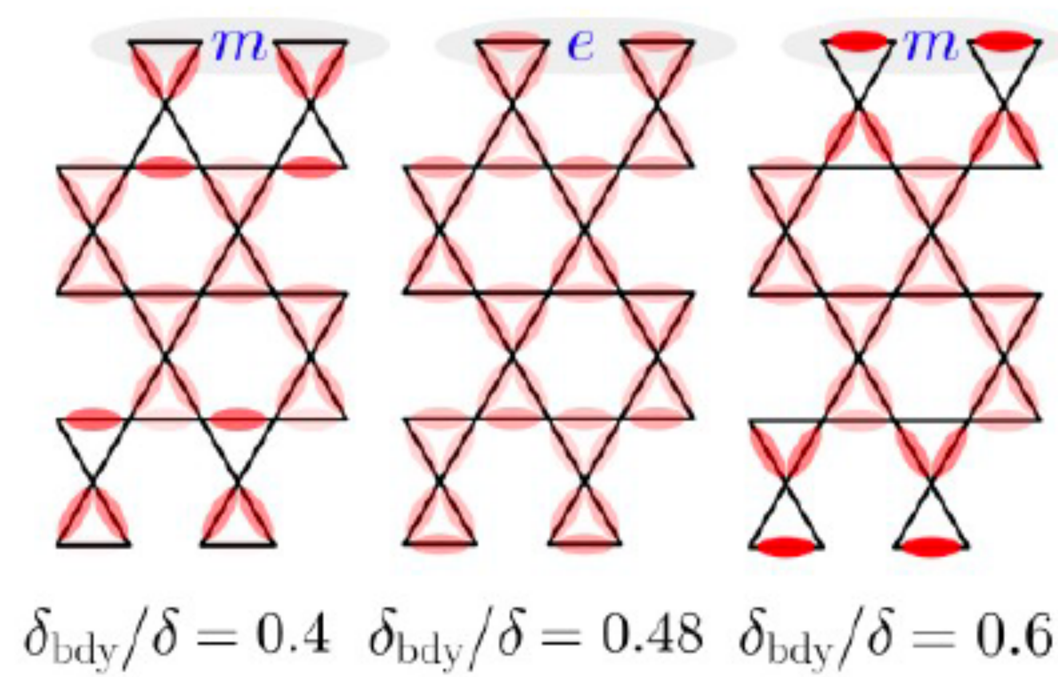


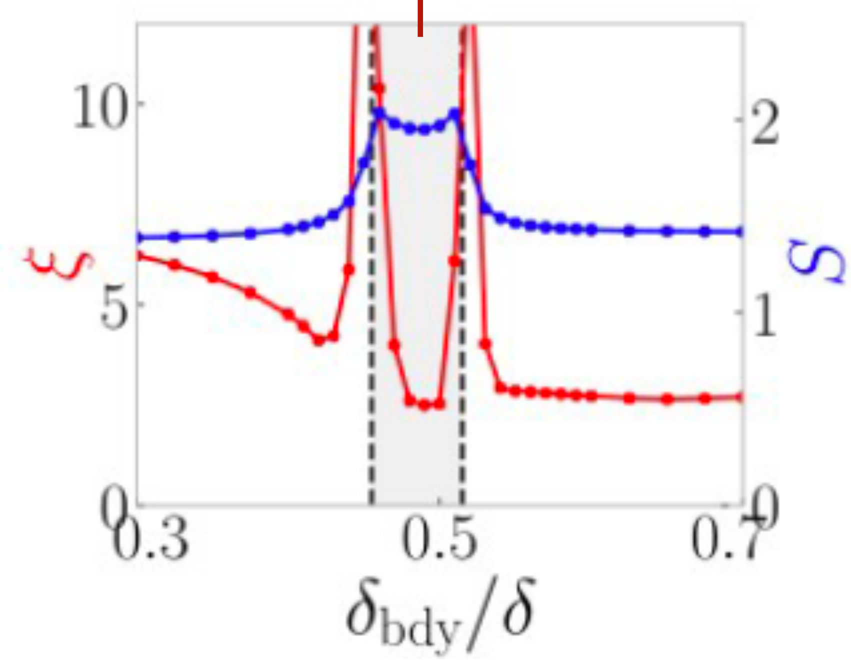


Tune boundaries
Two-fold degenerate

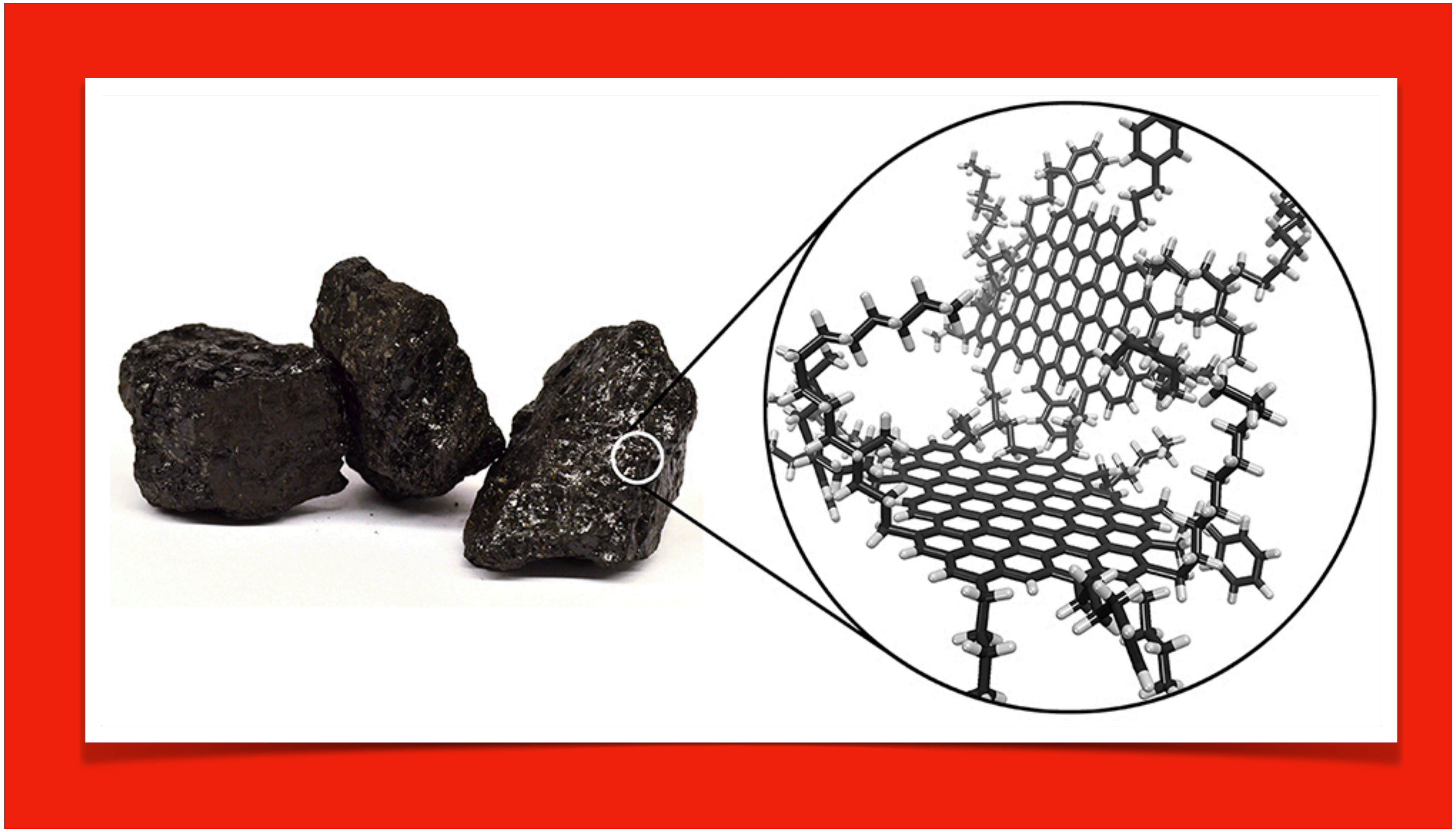
Trapping an anyon:







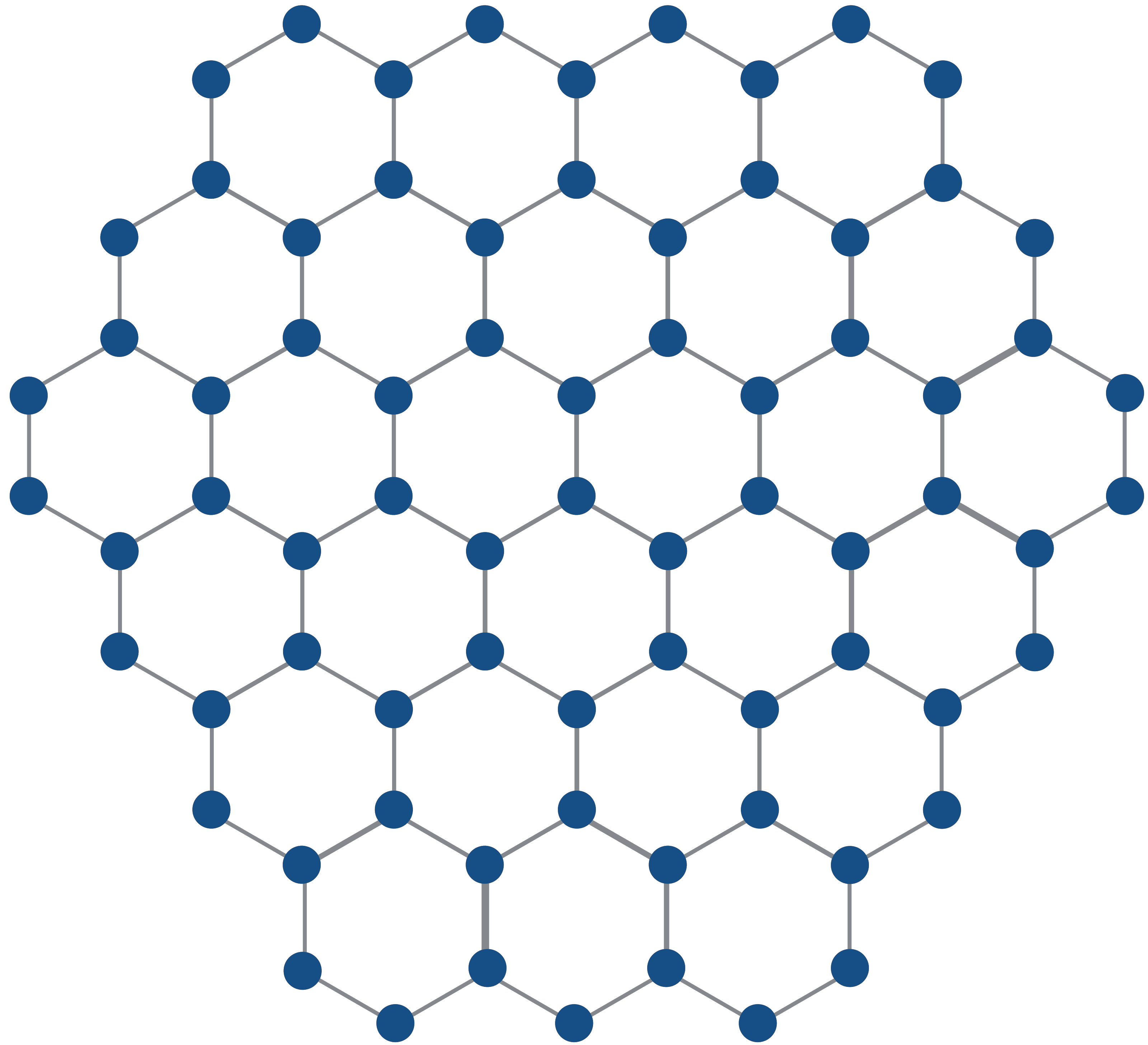
Boundary phase transition
On changing *detuning* δ



PHYSICAL REVIEW RESEARCH **2**, 023237 (2020)

Fractional Chern insulator states in twisted bilayer graphene: An analytical approach

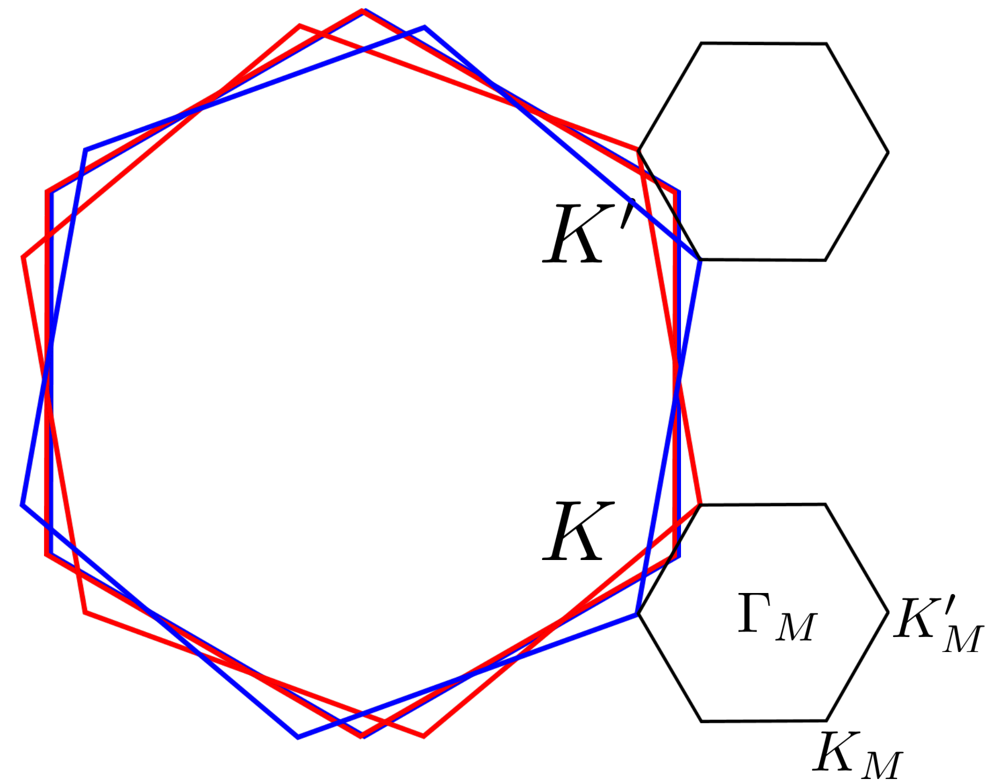
Patrick J. Ledwith, Grigory Tarnopolsky, Eslam Khalaf, and Ashvin Vishwanath
Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA



Twisted Bilayer
Monolayer
Bilayer

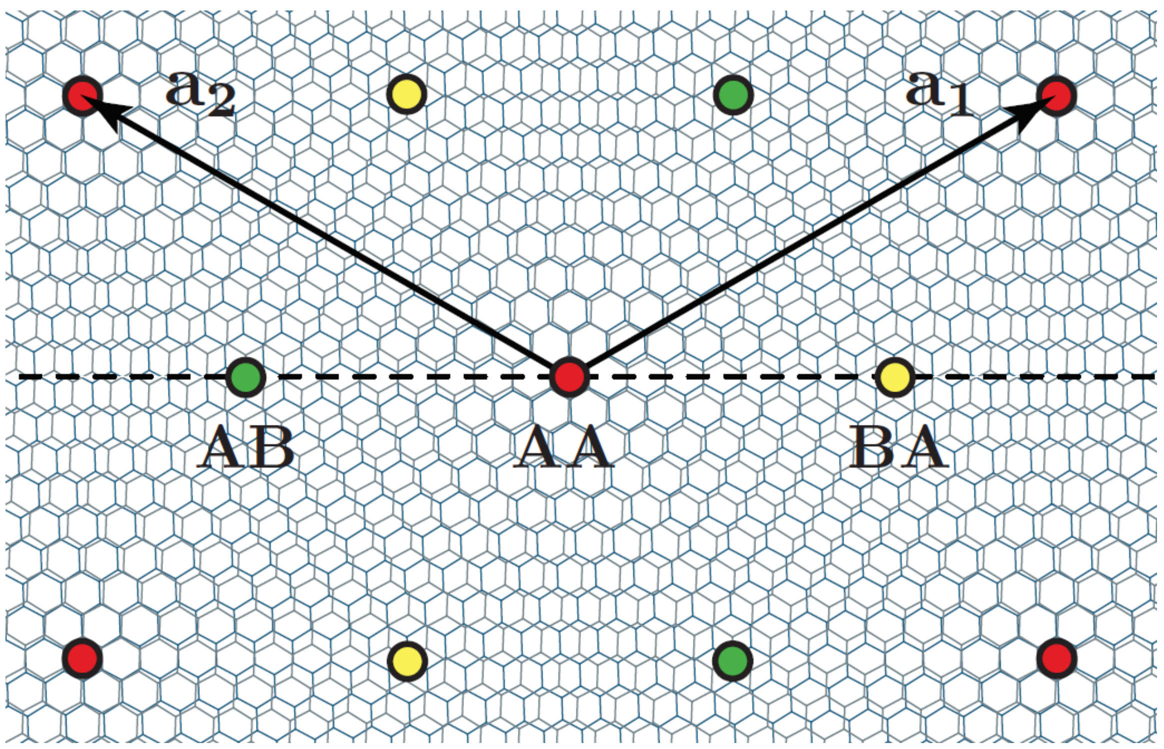
Continuum model

- Larger unit cell→ smaller BZone
- Bistrizer-Macdonald (BM) model (2011)



$$\mathcal{H}_K = \begin{pmatrix} -iv_F \boldsymbol{\sigma}_{\theta/2} \cdot \nabla & T(\boldsymbol{r}) \\ T^\dagger(\boldsymbol{r}) & -iv_F \boldsymbol{\sigma}_{-\theta/2} \cdot \nabla \end{pmatrix}_{12},$$

- Lattice relaxation: AB stacking favored to AA stacking (Carr et al. ’19, Nam, Koshino `17) $\implies w_0/w_1 \approx 0.7$

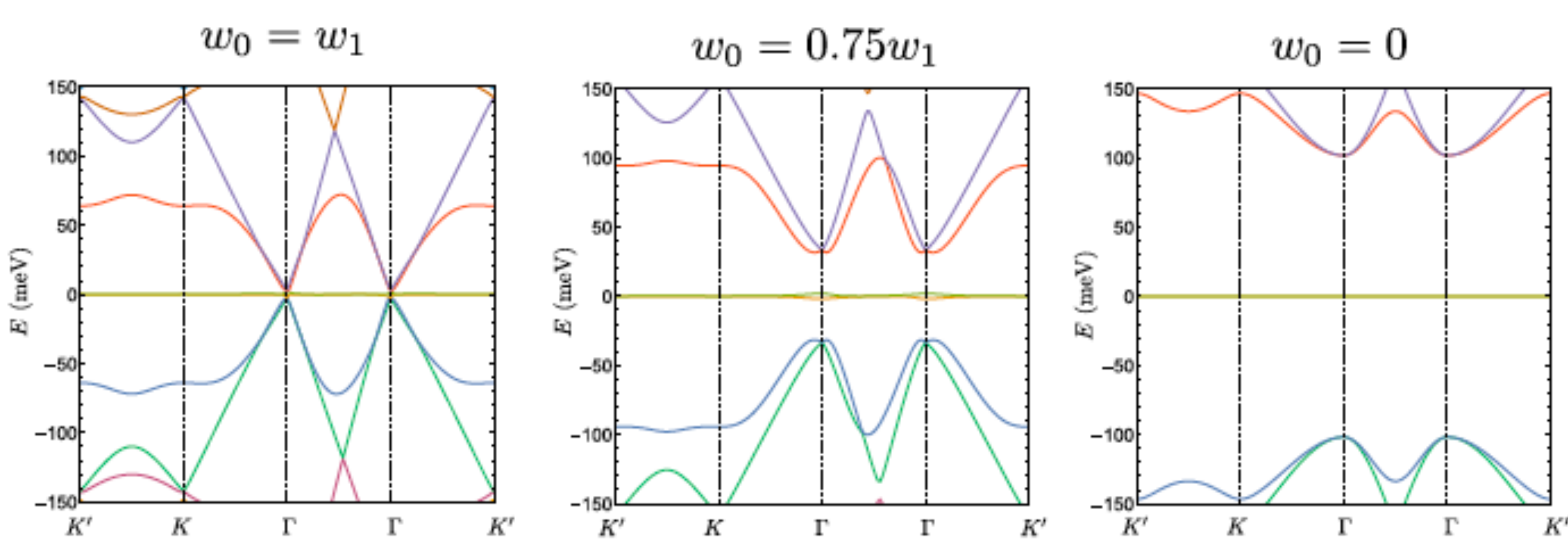


Chiral Model

Tarnopolski, Kruchkov, AV
PRL 2019

Switch off AA coupling. Only AB coupling

$$T(\boldsymbol{r}) = \begin{pmatrix} \cancel{w_0 U_0(\boldsymbol{r})} & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & \cancel{w_0 U_0(\boldsymbol{r})} \end{pmatrix}$$



Chiral Symmetry

$\{\sigma_z \otimes 1, \mathcal{H}\} = 0$

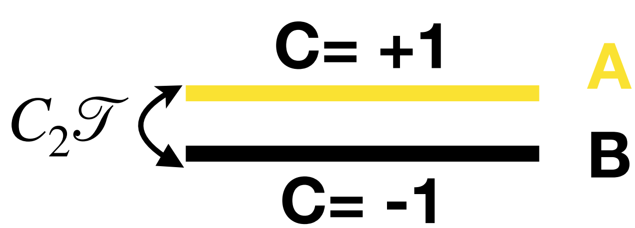
sublattice

Chiral Limit Wavefunctions

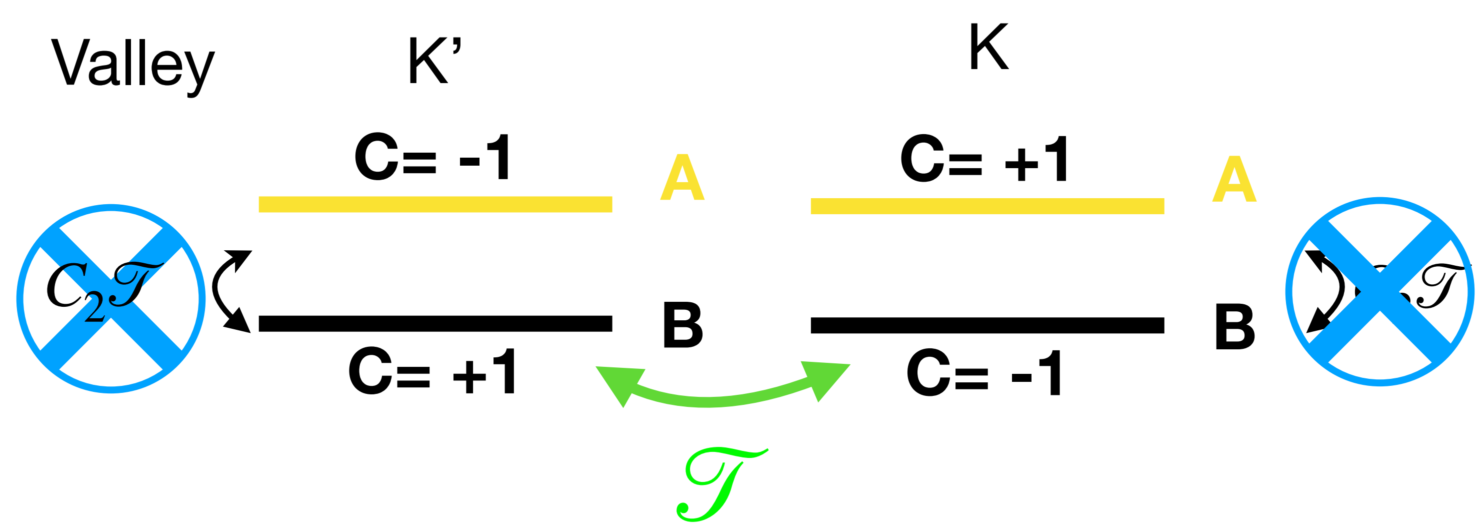
Many special properties:

(i) Chern number+ sublattice polarized

$$\psi_A(x,y) = \frac{\psi_K(x,y)}{\theta_1(\frac{z-z_0}{a_1}\,|\,\omega)} f(x+iy)$$



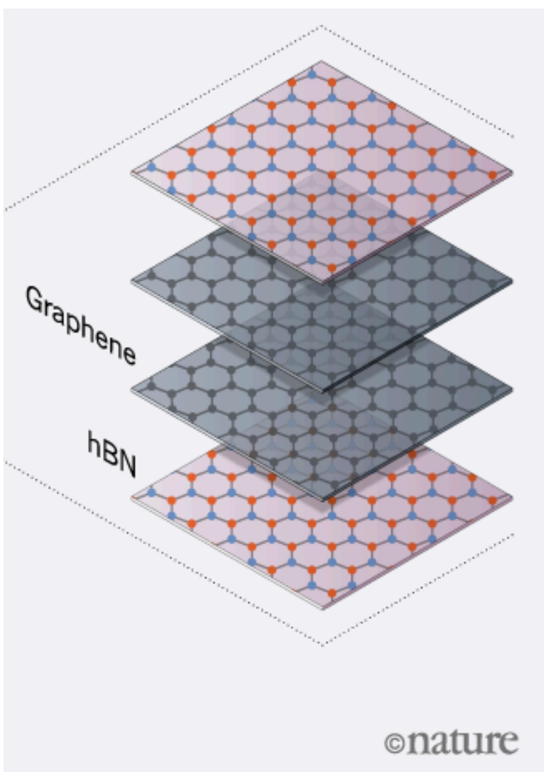
Chern =±1;
Sublattice polarized



Ledwith, Tarnopolsky, Khalaf, AV `20
Wang, Zheng, Millis, Cano `20
Becker, Embree, Wittsten, Zworski `20

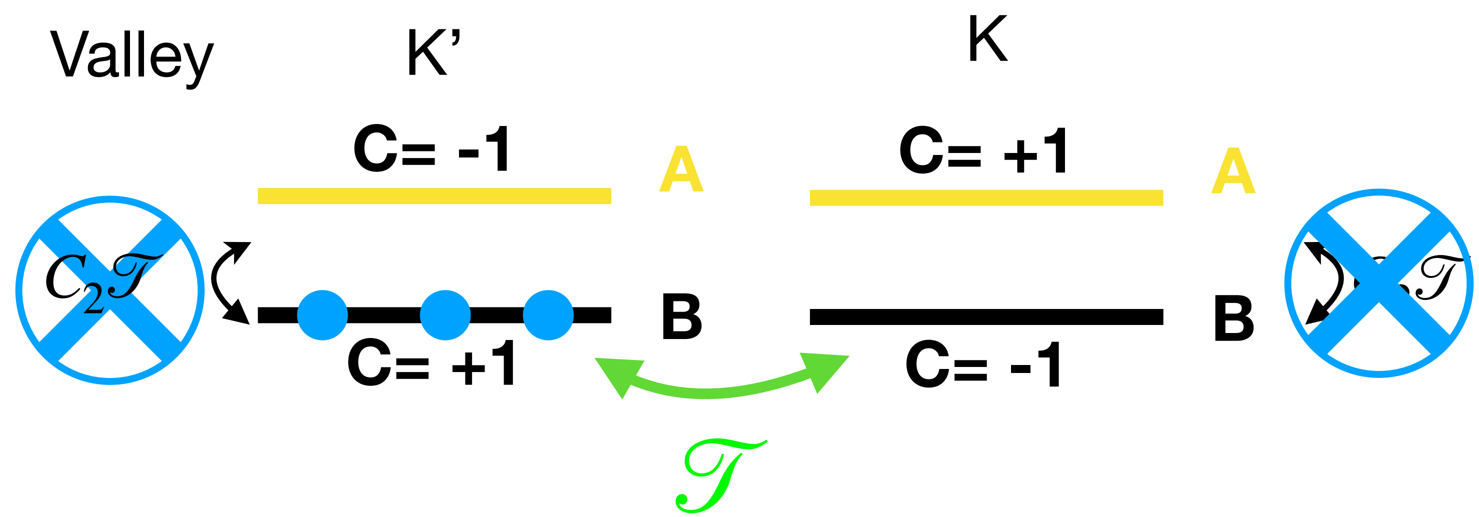
Intrinsic quantized anomalous Hall effect in a moiré heterostructure

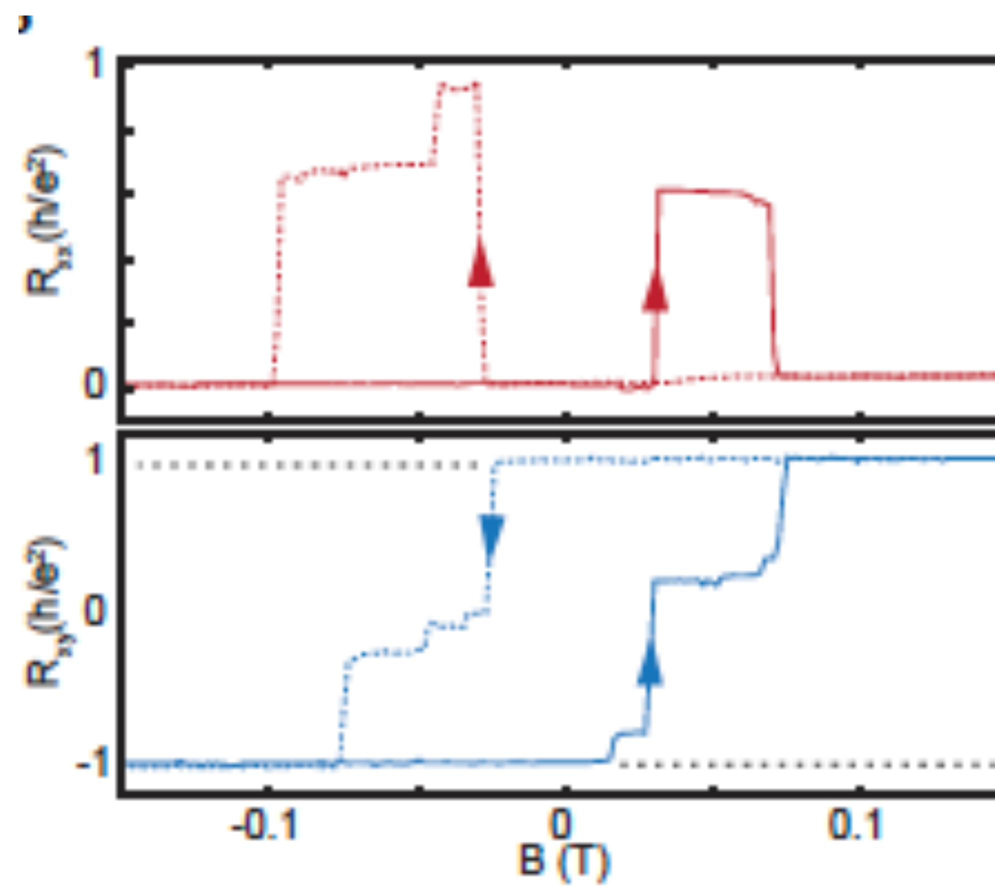
M. Serlin,[1,*] C. L. Tschirhart,[1,*] H. Polshyn,[1,*] Y. Zhang,[1] J. Zhu,[1] K. Watanabe,[2] T. Taniguchi,[2] L. Balents,[3] and A. F. Young[1,†]



<u>Aligned</u> h-BN substrate.
Break A-B symmetry

A. Sharpe…D. G-Gordon (2019)
M. Serlin… Young (2019)





Spontaneous Integer Hall at $\nu = 3$ - Fractional?

Chiral Limit Wavefunctions

Many special properties:

Ledwith, Tarnopolsky, Khalaf, AV `20

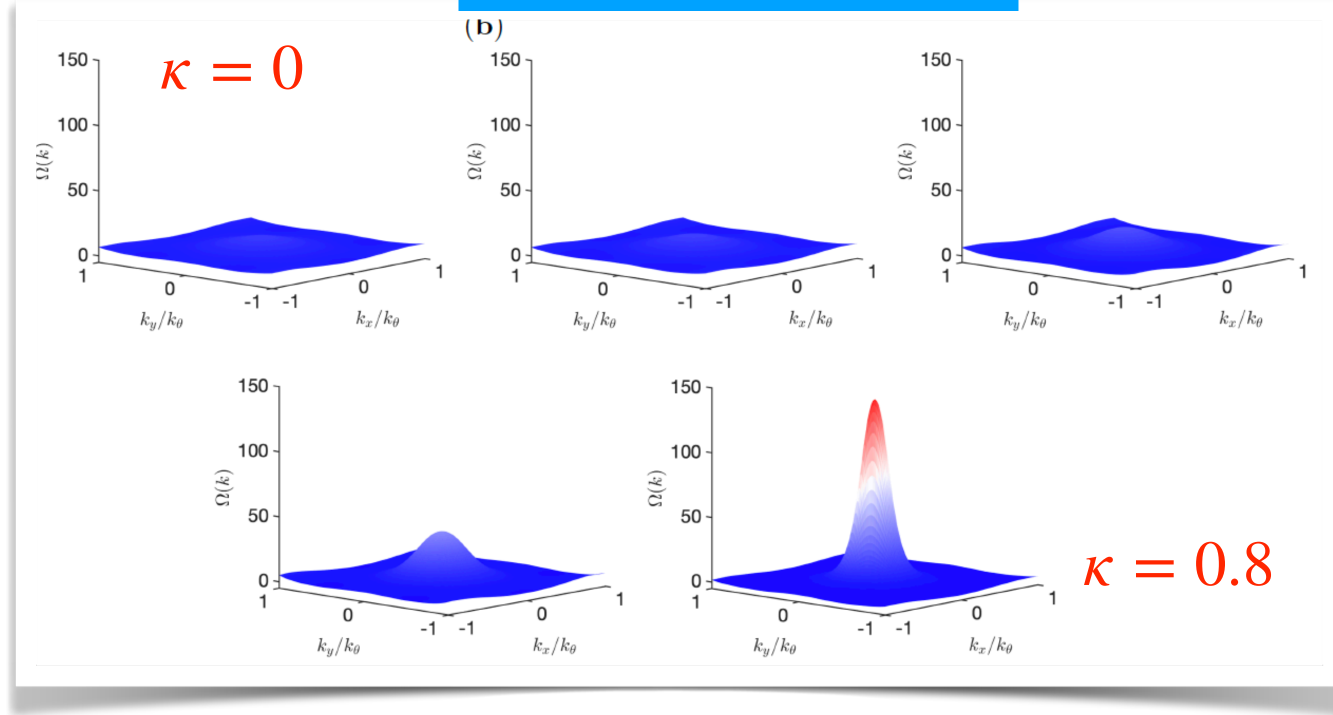
(i) Bloch wavefunctions - *analytic* in $k=k_x+ik_y$

$$u_{\mathbf{k}}(\mathbf{r}) = e^{-2\pi i r_2 k/b_2} \frac{\vartheta_1\left(\frac{z-z_0}{a_1} - \frac{k}{b_2}|\omega\right)}{\vartheta_1\left(\frac{z-z_0}{a_1}|\omega\right)} \psi_K(\mathbf{r})$$

(ii) nearly *uniform* Berry curvature

(iii) Isotropic ideal droplet condition -

Berry Curvature:



Ideal for Fractional Chern insulators

quantum metric:

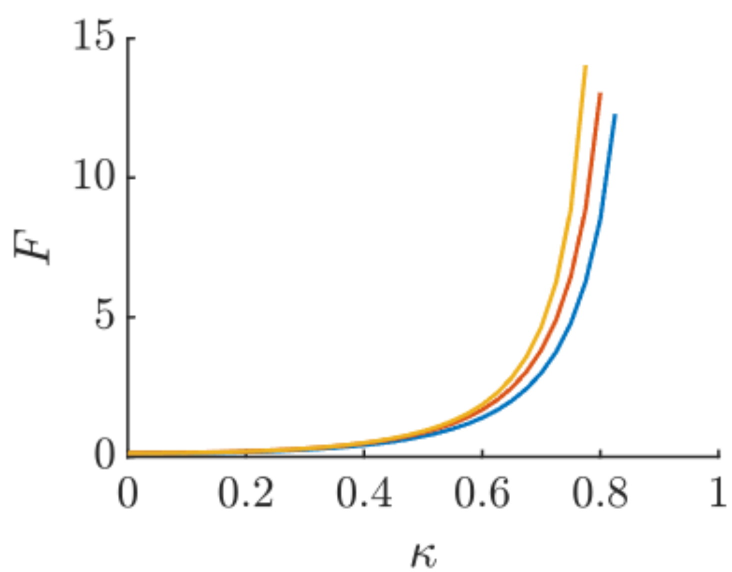
$$\eta(k) = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \Omega(k)$$

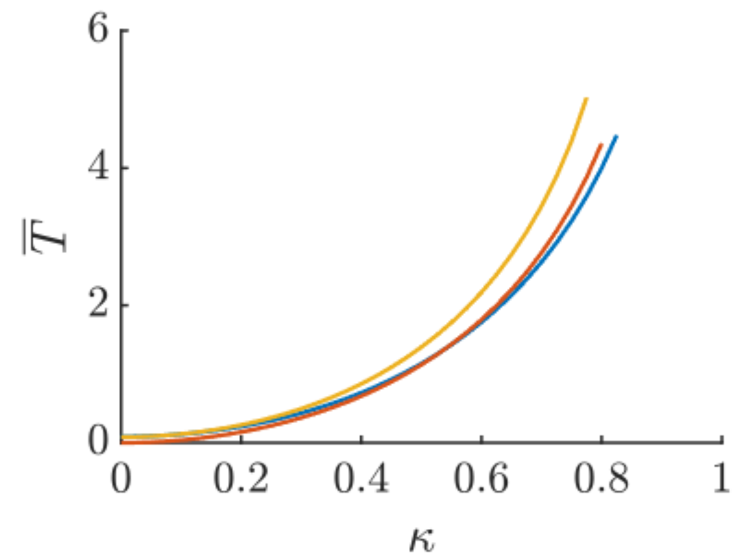
Roy Phys. Rev. B **90**, 165139 (2014)
Claassen et. al. Phys. Rev. Lett. **114**, 236802 (2015)
Jackson, Möller, Roy Nat. Comm. **6** 8629 (2015).
Ledwith, Tarnopolsky, Khalaf, AV `20

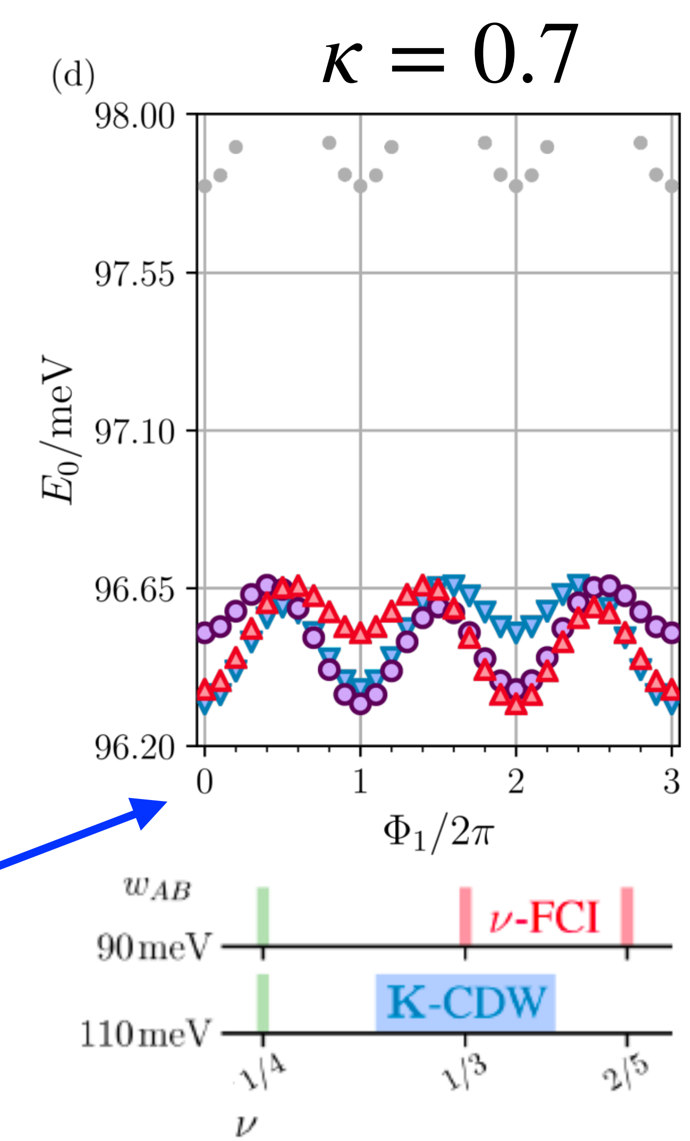
AA/BB Tunneling => non-ideal band geometry

- Berry curvature variation: $F = \left\langle \left(\frac{\Omega}{2\pi} - C \right)^2 \right\rangle_{\mathrm{BZ}}$

- Violation of ideal droplet condition: $\bar{T} = \langle \mathrm{tr}\, g - |\Omega| \rangle_{\mathrm{BZ}}$







Abouelkomsan, Liu, Bergholtz Phys. Rev. Lett. **124** 106803 (2020)
Repellin, Senthil Phys. Rev. Research. **2** 023238 (2020)
Wilhelm, Lang, Läuchli arXiv:2012.09829 (2020)

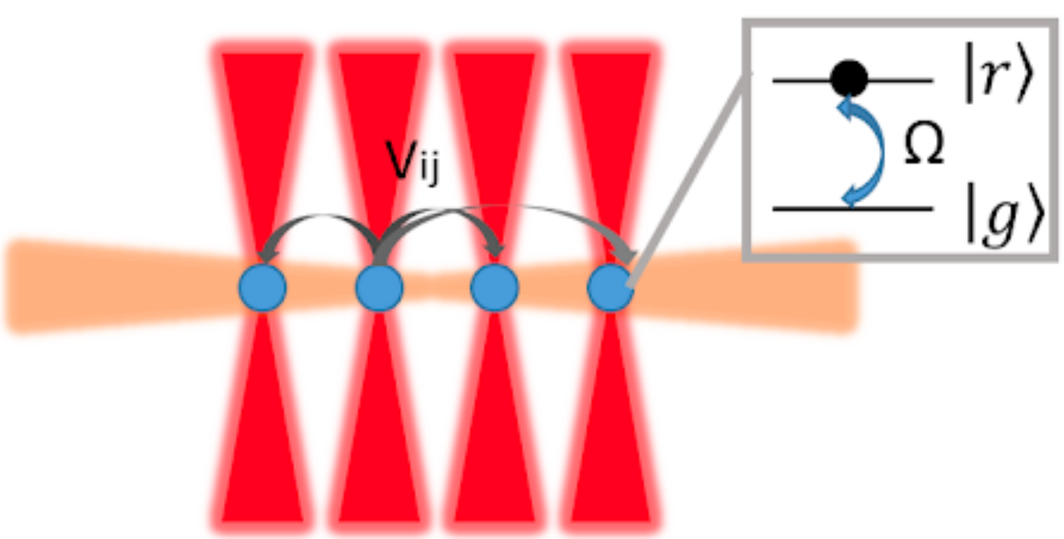
Conclusions:

Topological order, emergent gauge fields, non-local quantum entanglement could be more widespread than previously thought.

Need to explore new platforms and detection schemes.

<u>Rydbergs:</u>
Low symmetry
Long range interactions
Tunability

<u>Magic angle graphene:</u>
Remarkable (and mysterious) similarity to lowest Landau level



Bernien *et al.*