Flavor Magnetism and Superconductivity.

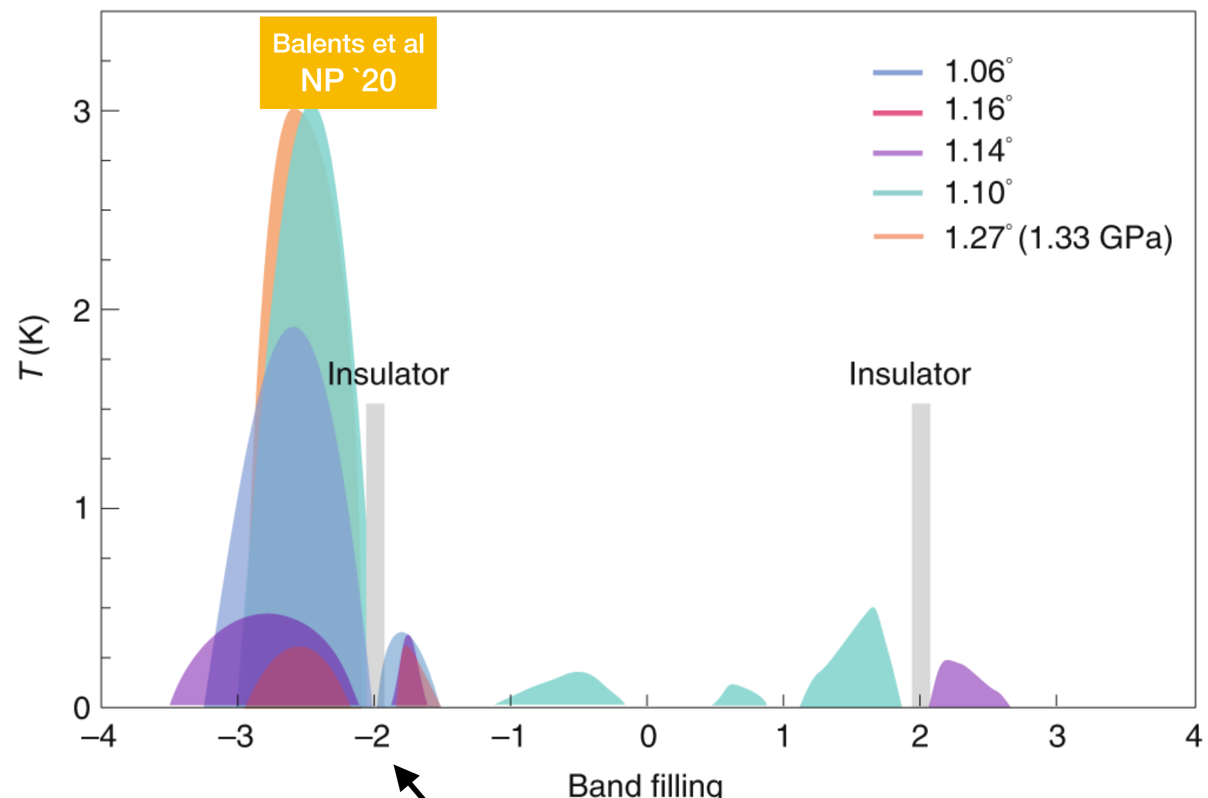
Theory of entwined orders in Magic Angle Graphene (s)

Ashvin Vishwanath

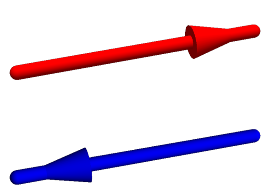
Lecture Notes for More Technical Details:

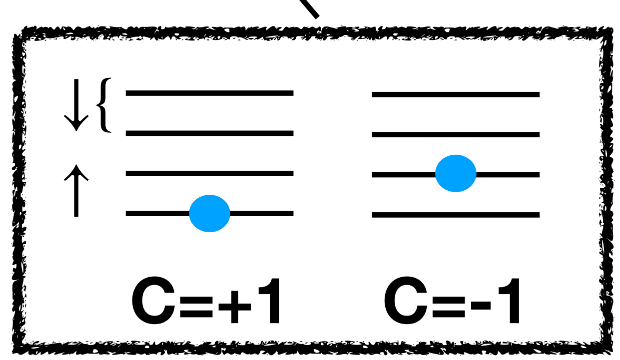
https://scholar.harvard.edu/avishwanath/teaching

Last Time:
Flat Band Topology and Flavor Magnetism

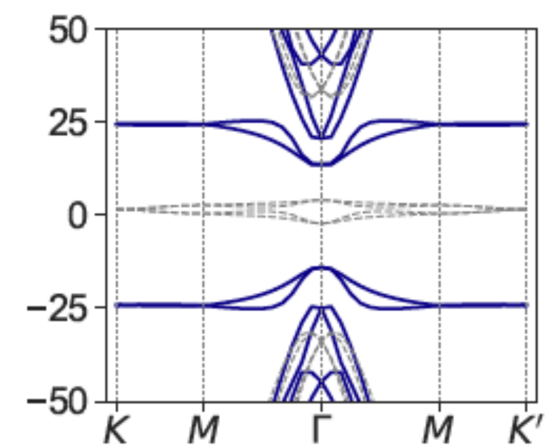


$$\hat{n}_+ = -\hat{n}_-$$





Adding Charge?



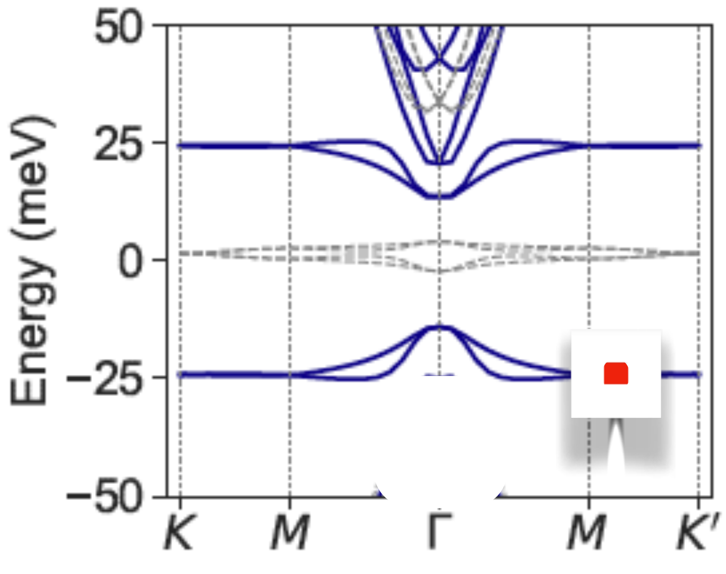
3. Superconductivity

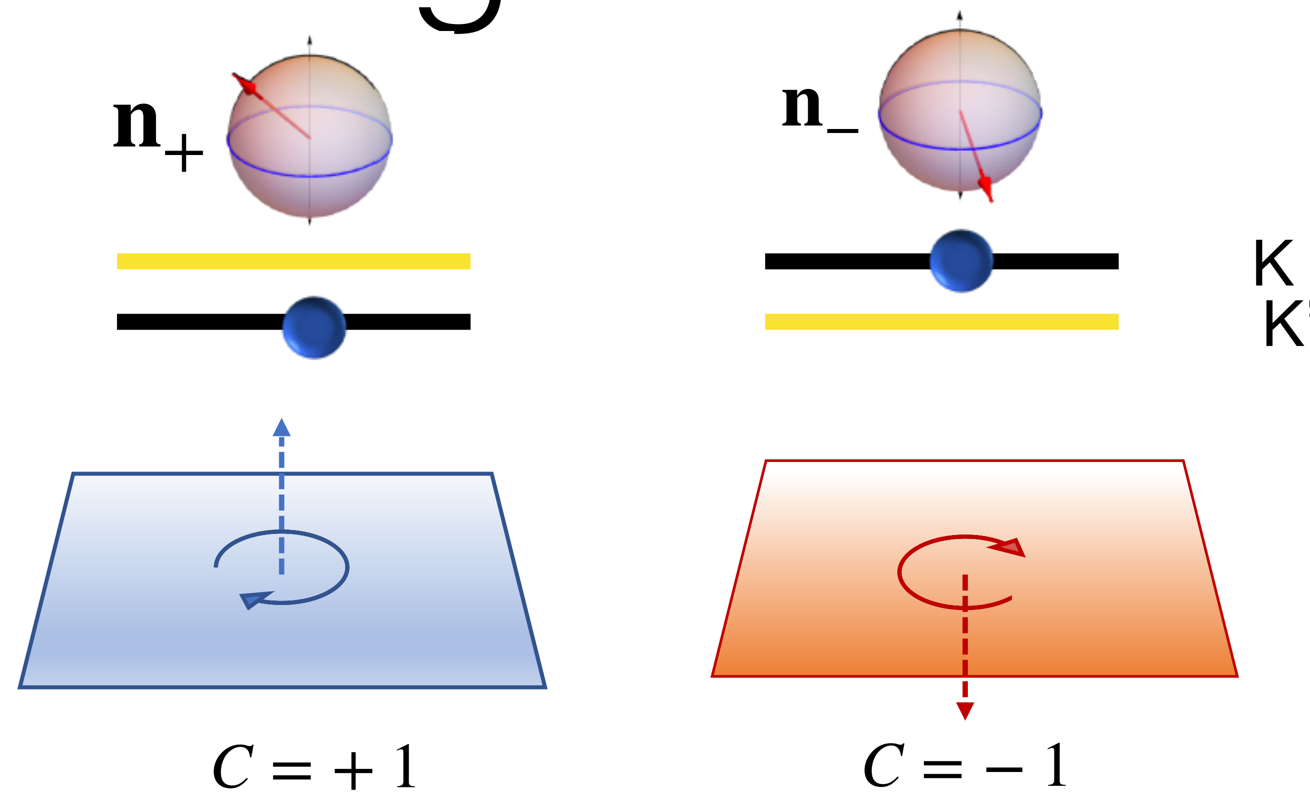
Khalaf, Chatterjee, Bultnick, Zaletel, AV arXiv:2004.00638

Symmetry constraints on superconductivity in twisted bilayer graphene: Fractional vortices, $4e$ condensates or non-unitary pairing

Eslam Khalaf, Patrick Ledwith, and Ashvin Vishwanath
Department of Physics, Harvard University, Cambridge, MA 02138

Adding Charge





Option 1 - introduce electronic degrees of freedom

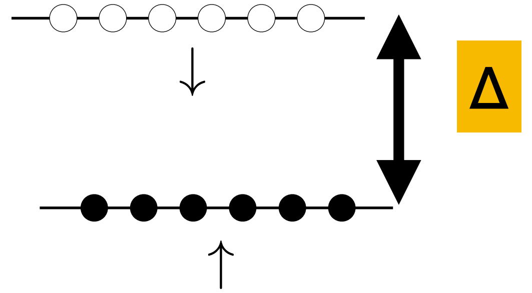
Option 2 - topological textures in $\mathbf{n}_+$ $\mathbf{n}_-$

- Topological sigma model - show topological textures in order parameter carry electric charge. "(Psuedo) spin-charge entanglement"

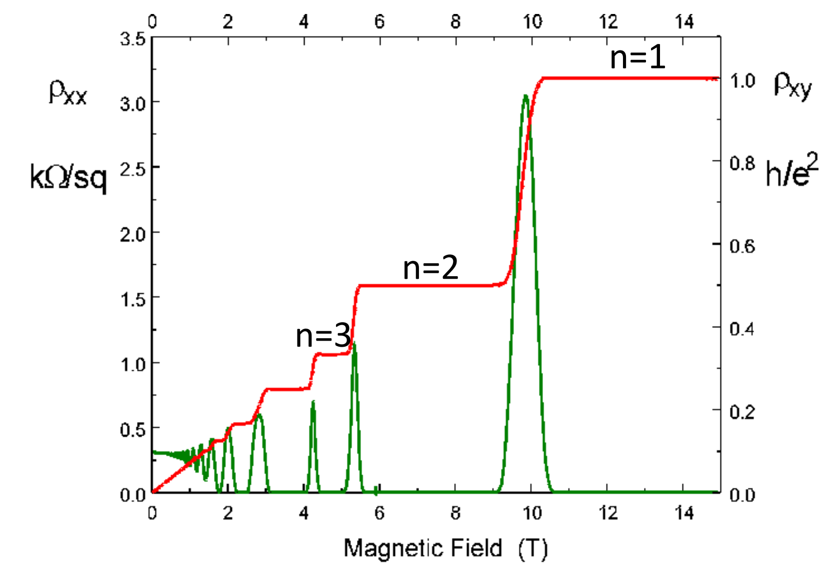
- Topological mechanism ofsuperconductivity?

also: Abanov and Wiegman, Grover & Senthil, Wang..Assad.
Here microscopic theory *with* Coulomb repulsion. **Khalaf, Chatterjee, Bultnick, Zaletel, AV arXiv:2004.00638**

ASIDE:Quantum Hall Ferromagnet and Skyrmions



A spin-degenerate Landau level



- Quantum Hall Ferromagnet - set $g_{Zeeman}=0$

Theory: Sondhi, Karlhede, Kivelson, Rezayi; Lee & Kane

$$\mathcal{H}_{\text{int}} = \frac{1}{2A}\sum_{\boldsymbol{q}} V_{\boldsymbol{q}}\delta\rho_{\boldsymbol{q}}\delta\rho_{-\boldsymbol{q}},$$

Ground state - ferromagnet.

Projected density operators and interaction: $V_q = e^{-q^2 l_B^2/2}\dfrac{e^2}{|q|}$

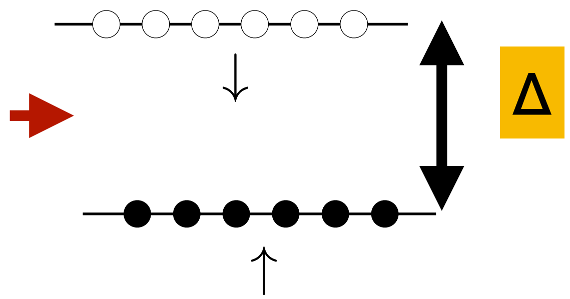
Charge Excitations: $V(r\rightarrow\infty) = \dfrac{e^2}{r};\ V(r\rightarrow 0) = \sqrt{\dfrac{\pi}{2}}E_c$ **So** $\Delta = \sqrt{\dfrac{\pi}{2}}E_c \sim e^2/l_B$

Also spin waves: $E = \dfrac{\rho_s}{2}(\nabla\hat{n})^2$ Where $\rho_s = \dfrac{\Delta}{16\pi}$

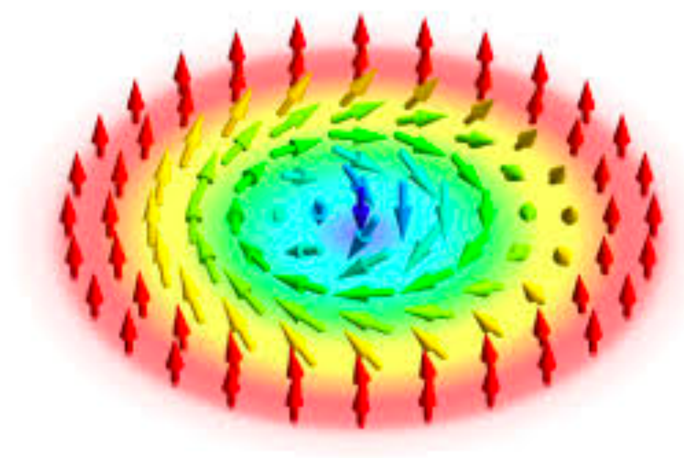
Girvin cond-mat/9907002

Quantum Hall Ferromagnet and Skyrmions

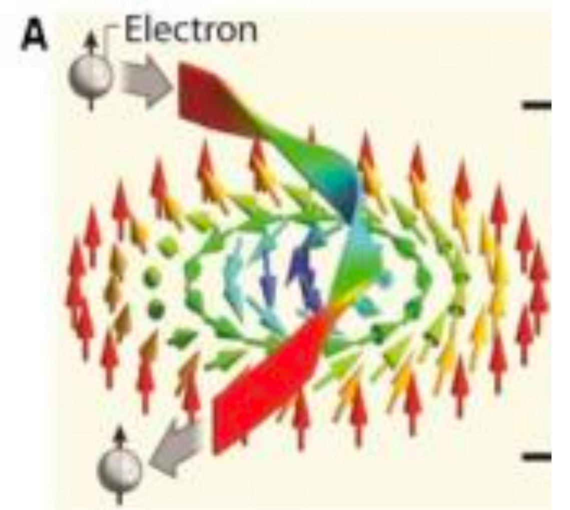
ESkx $=8\pi\rho$



A spin-degenerate Landau level



Schütte&Garst



A. Rosch

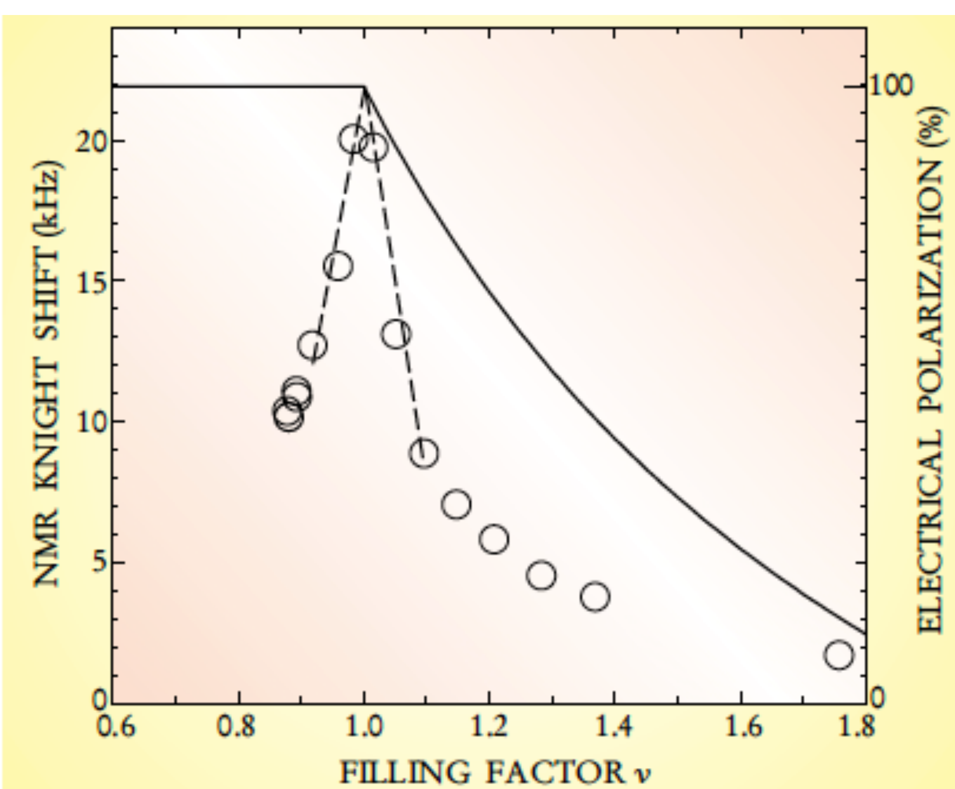
- Quantum Hall Ferromagnet

Electric charge carried by *Skyrmions*.

$$\rho_c = \sigma_{xy}\frac{e}{4\pi}\hat{n}\cdot\partial_x\hat{n}\times\partial_y\hat{n}$$

1

Theory: Sondhi, Karlhede, Kivelson, Rezayi; Lee & Kane
Experiments: NMR - Barrett et al.



$$\mathbf{E}_{\mathbf{Skx}} = 8\pi\rho_s = \frac{\Delta}{2}$$

Electron -> ferromagnet
Ferromagnet -> electron

[*topological* σ-model]

Tony Skyrme

A UNIFIED FIELD THEORY OF MESONS AND BARYONS

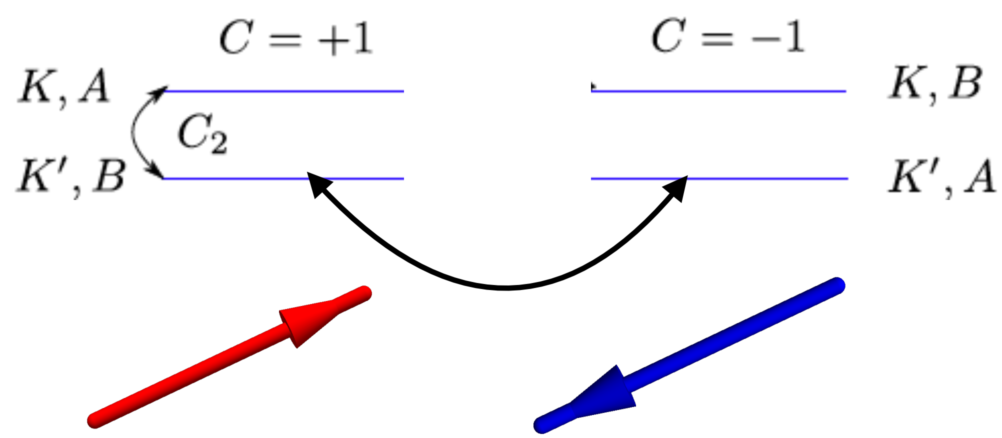
T. H. R. SKYRME †

A.E.R.E., Harwell, England

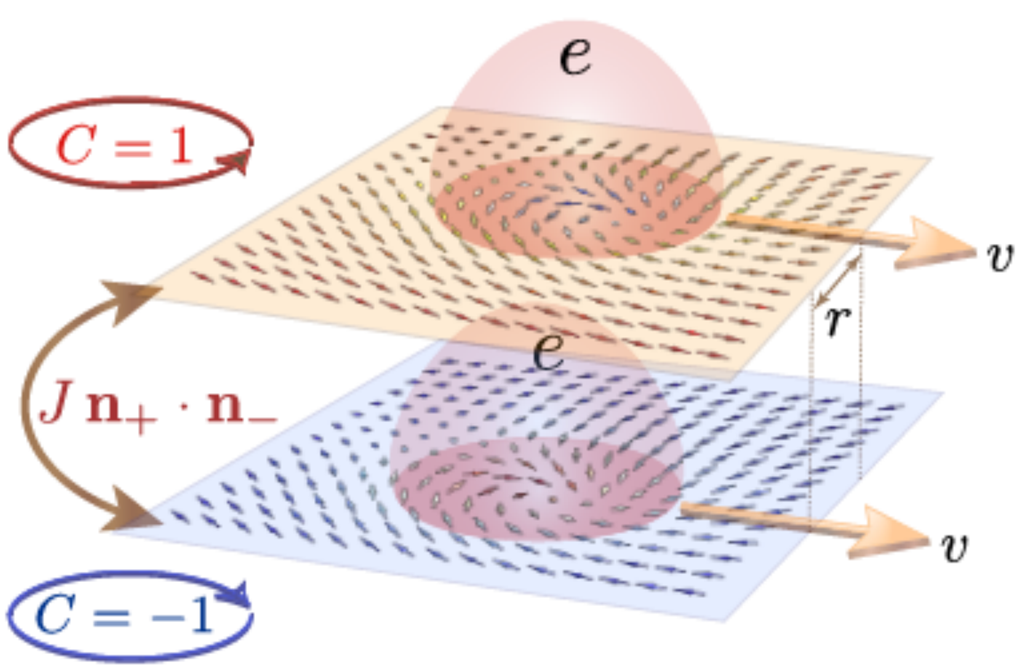
Nuclear Physics **31** (1962) 556—569;

We are indebted to Mr. A. J. Leggatt for carrying out the calculations reported in sect. **3**, while a vacation student at A.E.R.E.

Skyrmions in Magic Angle Graphene?



Antiferro-psuedospin coupling $J\mathbf{n}_+ \cdot \mathbf{n}_-$ **J~h²/V~1-2 meV**



Charge 2e Skyrmion
Boson
Pairing due to `J'

Repelled by Coulomb
But attracted by J

An all electron mechanism?

Topological σ model

•Non-linear sigma model for $\boldsymbol{n}_\pm$ (following Moon *et al.* 1995)

•Antiferromagnetic sector: integrate out ferromagnetic fluctuations.

$\boldsymbol{n} = \boldsymbol{n}_+ = -\boldsymbol{n}_-$

$$\mathcal{L}[\mathbf{n}] = \rho_n (\nabla \mathbf{n})^2 + \chi_n (\partial_t \mathbf{n})^2 - \mu \frac{2e}{4\pi} \mathbf{n} \cdot \partial_x \mathbf{n} \times \partial_y \mathbf{n} + V_{Coulomb}[\mathbf{n}]$$

- **Note**:
  - \`antiferromagnetic' sigma model - no Berry phase term, quadratic time derivative.
  - Topological textures carry charge - couples to μ
  - Coulomb interaction included - interaction between topological densities.

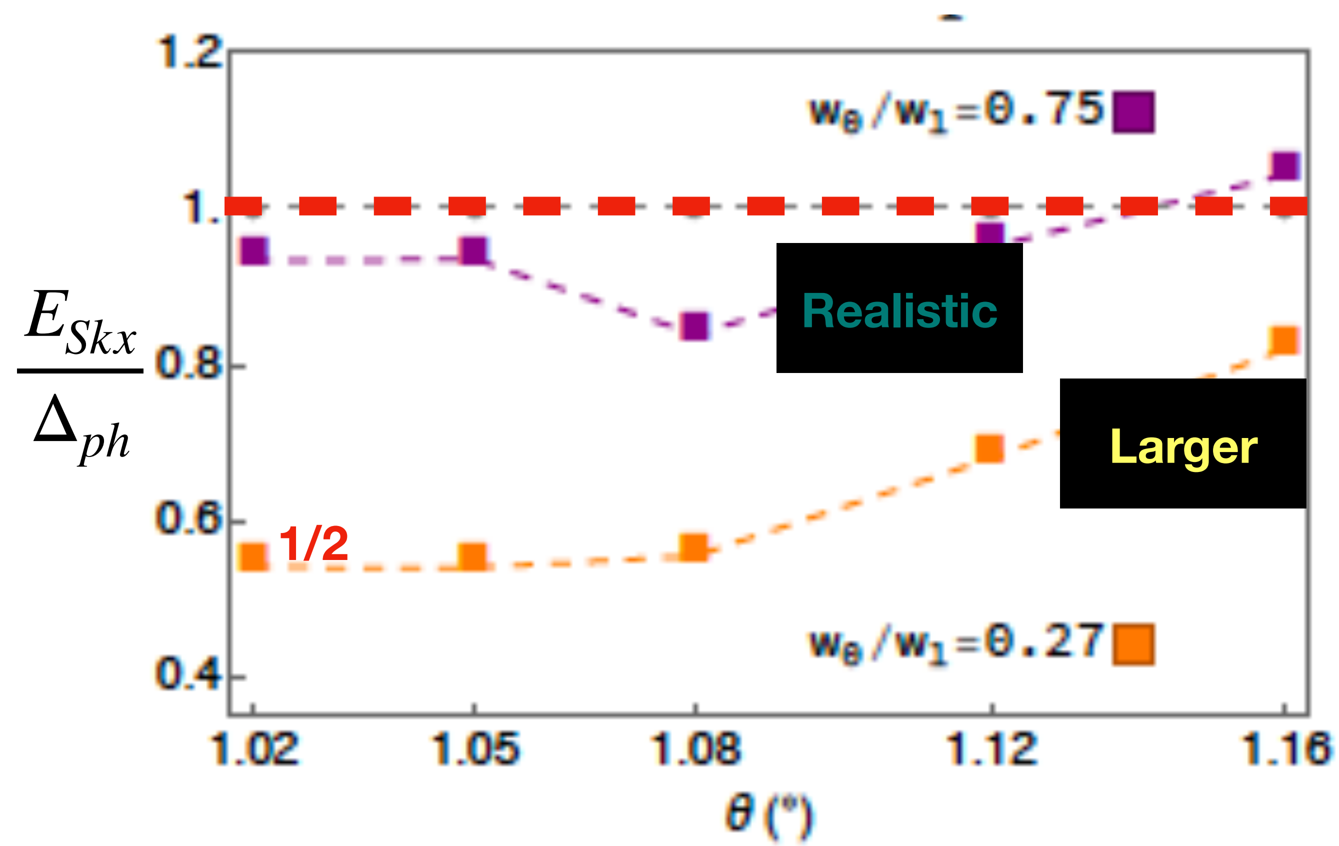
Doping the Insulator

- Use microscopically obtained values of parameters to:
    - Compare energy of skyrmions to 1 particle band gap
    - `Inertia' of skyrmions- sets condensation temperature.

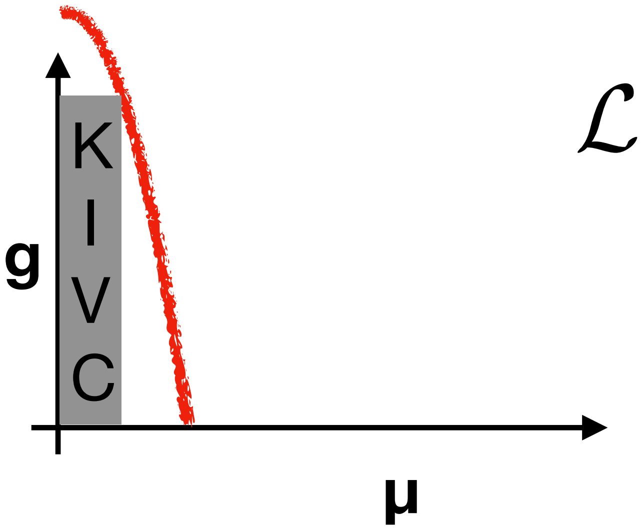
- Elastic energy $E_{\text{sk,pair}} = 8\pi\rho_{\text{ps}}$ vs Δ_{PH}

$$E = \frac{\rho}{2}\left[(\nabla n_+)^2 + (\nabla n_-)^2\right] + J n_+ \cdot n_-$$

*Neglects anisotropies, fermion dispersion

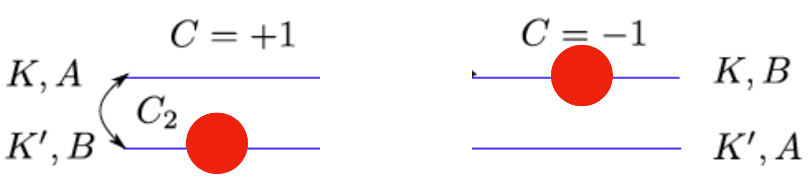


Phase Diagram & Stiffness



$$\mathcal{L}[\mathbf{n}] = \rho_n(\nabla\mathbf{n})^2 + \chi_n(\partial_t\mathbf{n})^2 - \mu\frac{2e}{4\pi}\mathbf{n}\cdot\partial_x\mathbf{n}\times\partial_y\mathbf{n} + V_{Coulomb}[\mathbf{n}]$$

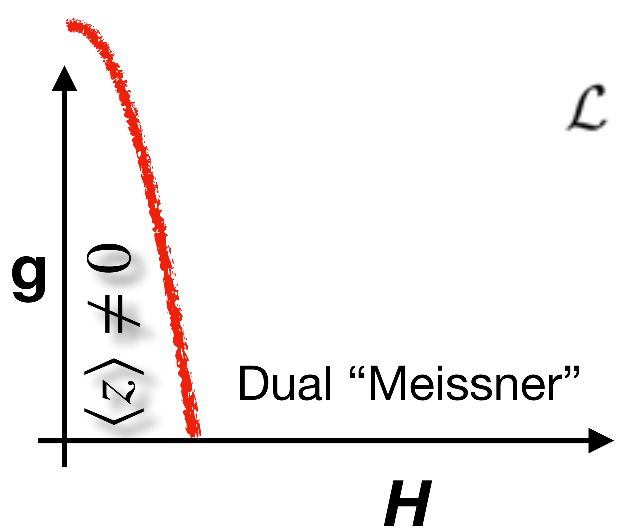
A Dual Representation:



$$|\psi\rangle = z_1|K,A\rangle + z_2|K',B\rangle \qquad z=(z_1,z_2), \quad \boldsymbol{n} = z^\dagger\boldsymbol{\sigma}z,$$

Rewrite in terms of CP1 fields,
Note ambiguity in overall phase.

$$\mathcal{L} = \frac{\Lambda}{g}|(\partial_\mu - ia_\mu)z|^2 + \mu\frac{b}{\pi} \quad \textbf{where} \quad \nabla\times a = b = \frac{1}{2}\hat{n}\cdot\partial_x\hat{n}\times\partial_y\hat{n}$$

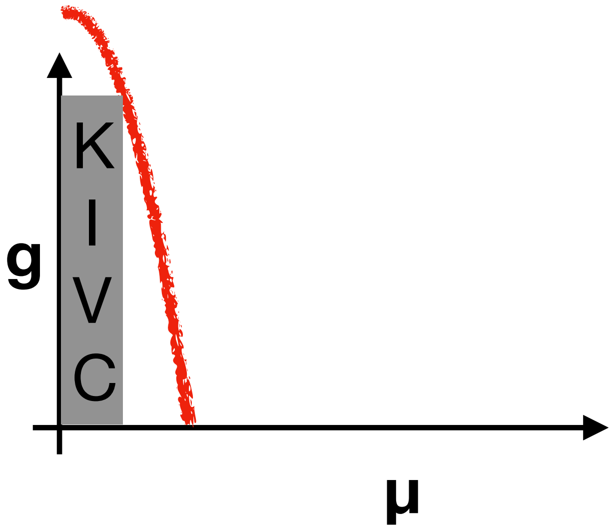


Superfluid Tc? Coefficient of $(\partial_t a)^2/2\rho_s$

EK, Chatterjee, Bultinck, Zaletel, Vishwanath arXiv:2004.00638

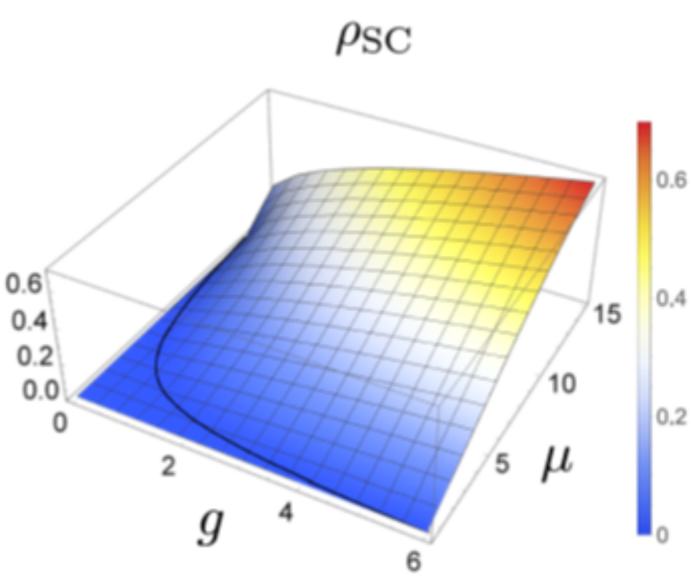
Solve using large-N CP$_N$

Phase Diagram & Stiffness

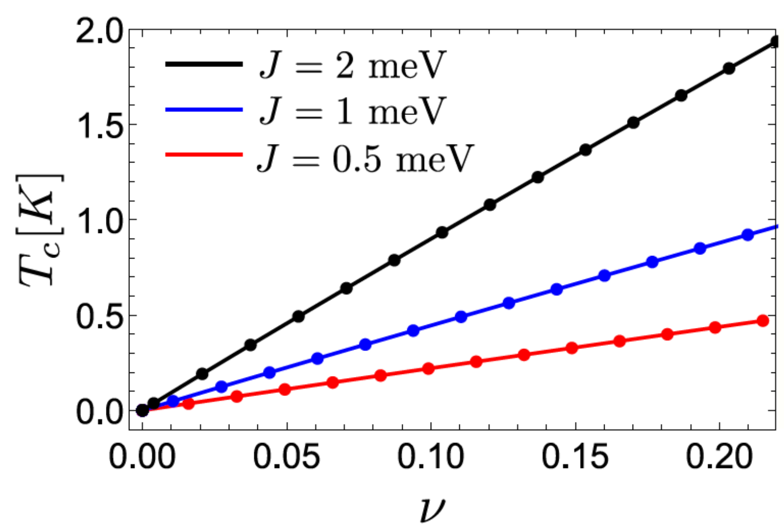


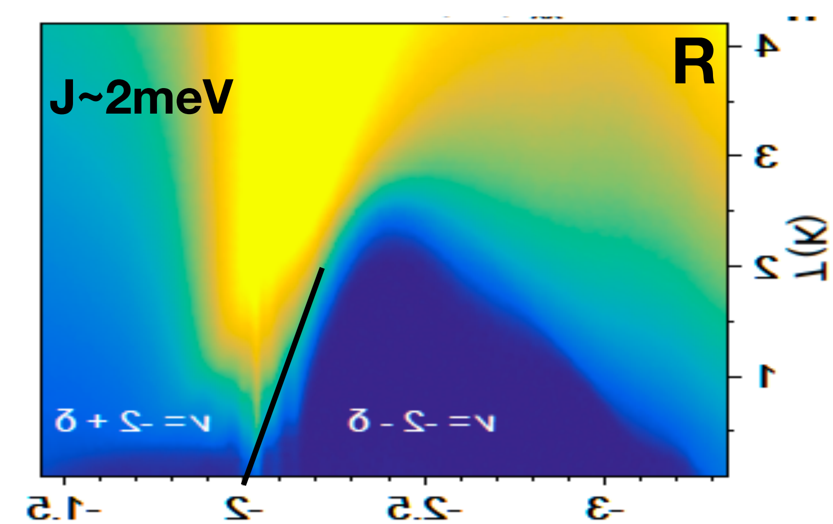
Solve using large-N CP_N

$$\mathcal{L} = \frac{\Lambda}{g}|(\partial_\mu - i a_\mu) z|^2 + \mu \frac{b}{\pi}$$



$$T_c = \frac{3\nu A_M J}{4N k_B} + O(\nu^2)$$





J. Park, Y. Cao … Pablo `20

Numerical DMRG evidence - Shubhayu's talk

Chatterjee *et al.* arxiv:2010.01144

EK, Chatterjee, Bultinck, Zaletel, Vishwanath arXiv:2004.00638

Skyrmion Properties

- Effective Mass

Skyrmion -antiskyrmion pair.

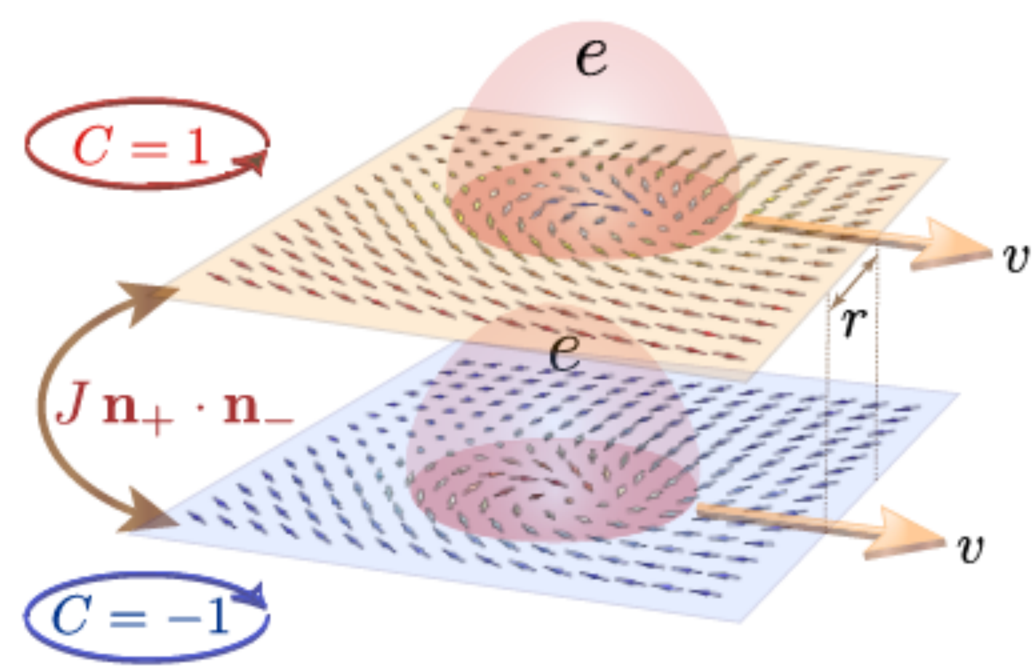
Individually - experience opposite Magnus dynamics.
Together - inertial dynamics.
What is effective mass?

$$\mathscr{L} = \frac{1}{2}(r_1 \wedge \dot{r}_1 - r_2 \wedge \dot{r}_2) - V(r_1 - r_2)$$

$$\mathscr{L}[R = \frac{r_1 + r_2}{2}, r = r_1 - r_2] = r \wedge \dot{R} - V(r)$$

$$V(r) = \frac{J}{2} r^2$$

$$\frac{1}{m} = \frac{\partial^2 E}{\partial P^2} \propto \frac{\partial^2 V}{\partial r^2} \sim J$$



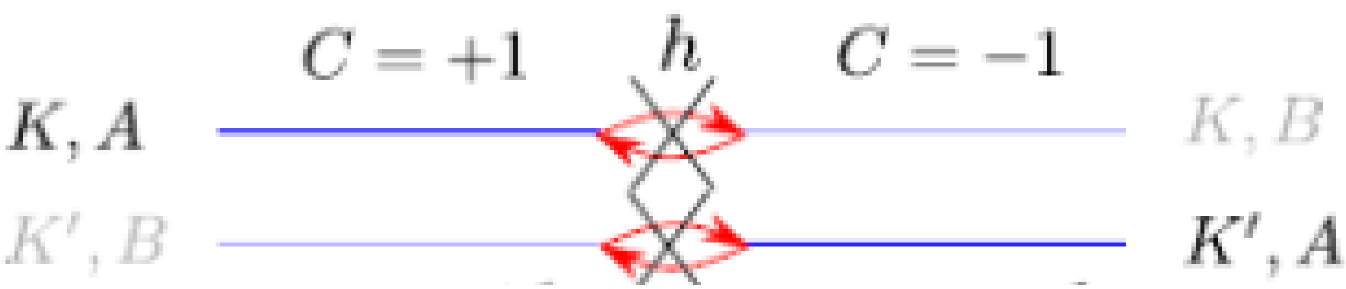
Sets superconductor Tc on doping

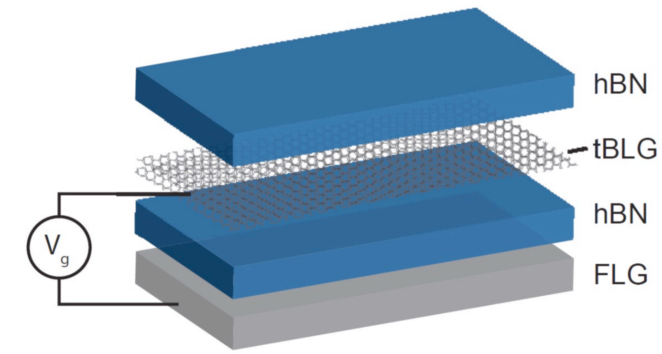
$$T_c \sim \nu J$$

- Khalaf, Chatterjee, Bultinc, Zaletel, AV arxiv: 2004.00638

Essential Ingredients for Superconductivity?

- $C_2\mathcal{T}$ symmetry crucial for superconductivity
- Sublattice potential *suppresses* the coupling J





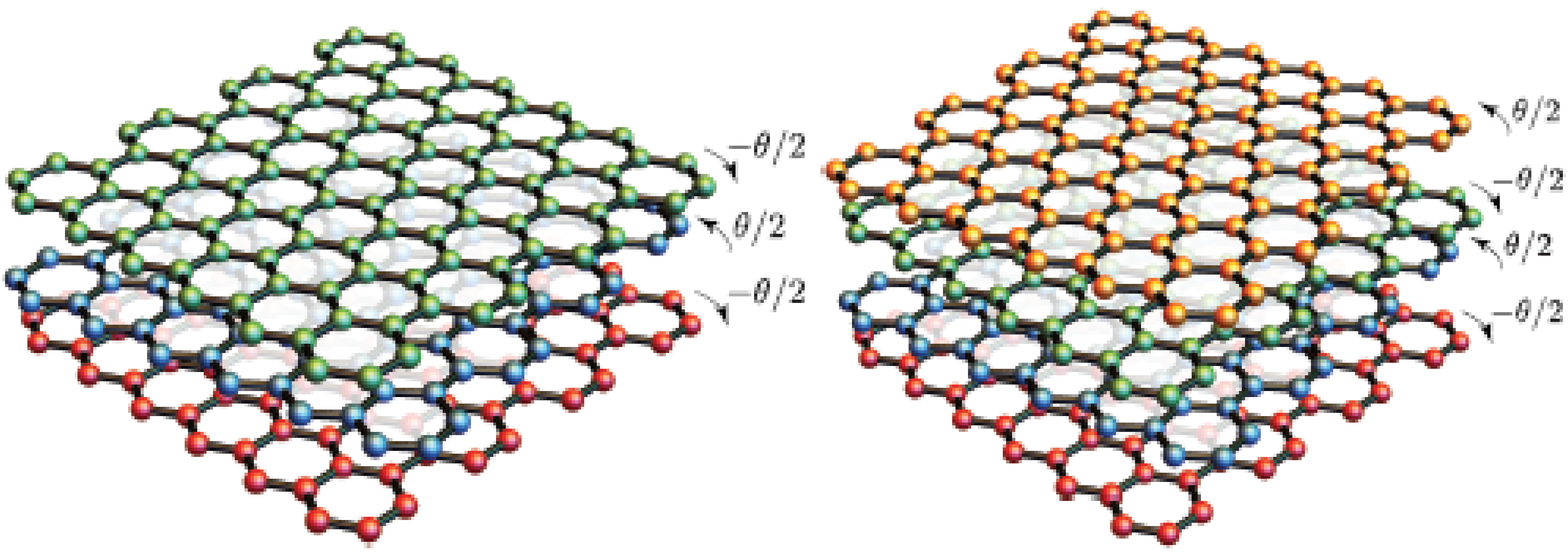
- Twisted bilayer graphene aligned on hBN → no superconductivity (Sharpe et al. Science 19, Serlin et al. Science 20)

- Relatively *few* Moire materials apart from magic angle graphene with this symmetry.

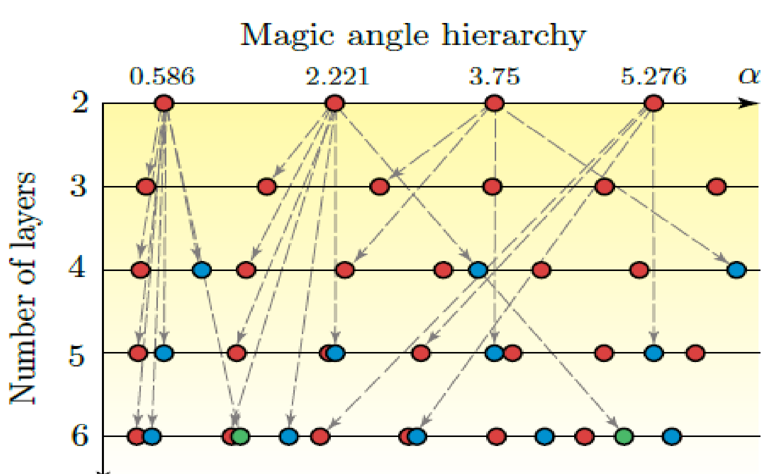
New platforms for Superconductivity?

- Other systems with C2 symmetry

Alternating twist sandwich



Eslam Khalaf

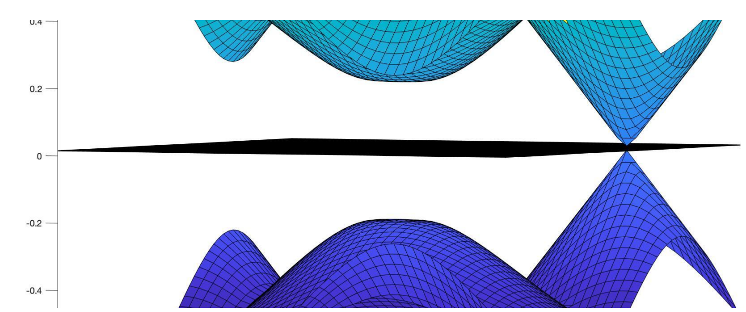


$\sqrt{2}$

$\varphi = \frac{1+\sqrt{5}}{2}$

Larger magic angles!

Magic angle = $\sqrt{2}\,1.1^o \sim 1.55^o$



Flat band+Dirac

Carr et al. stability `19

Khalaf, Kruchkov, Tarnopolsky, AV, 1901.10485

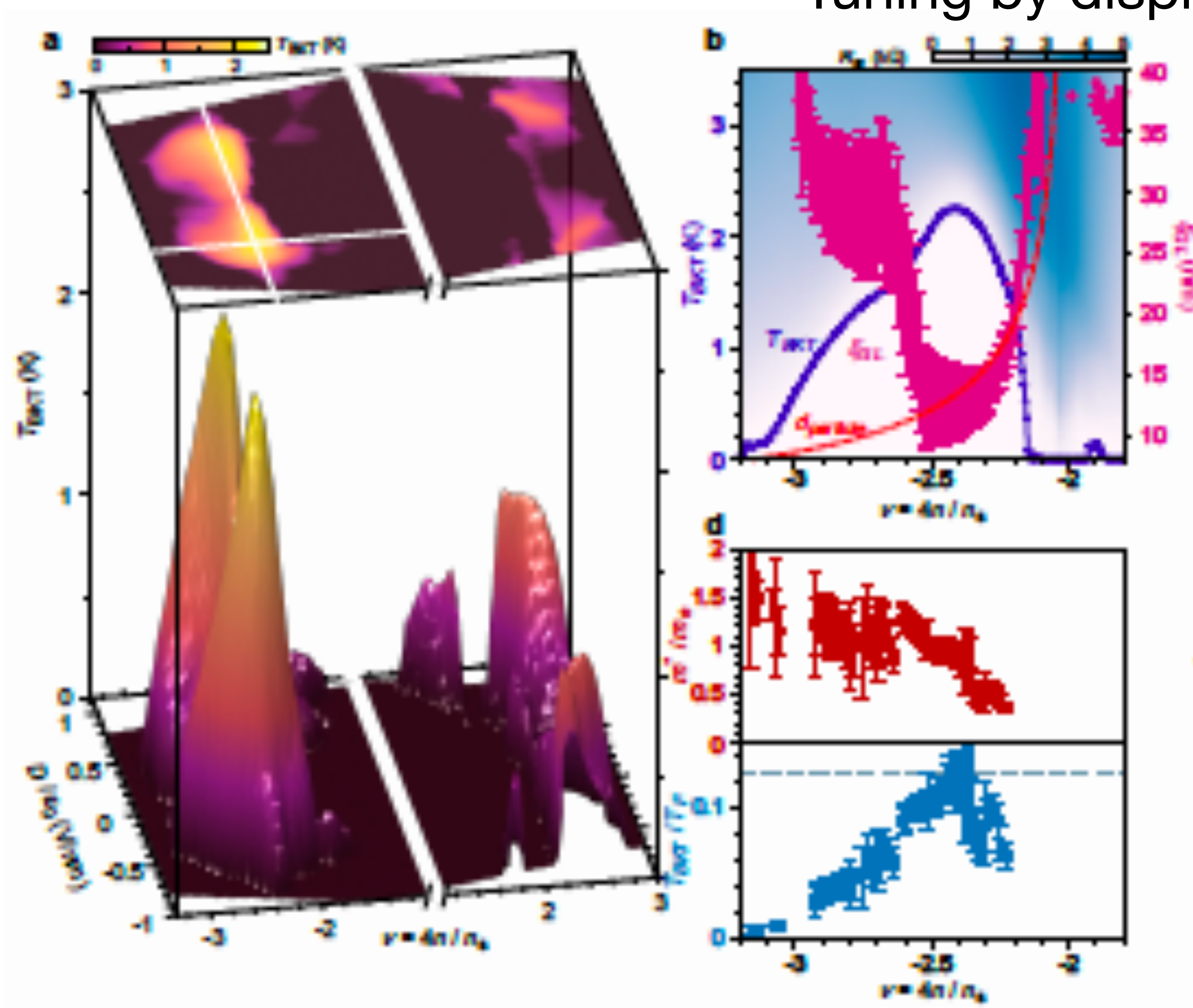
Matthias' Rules for Superconductivity

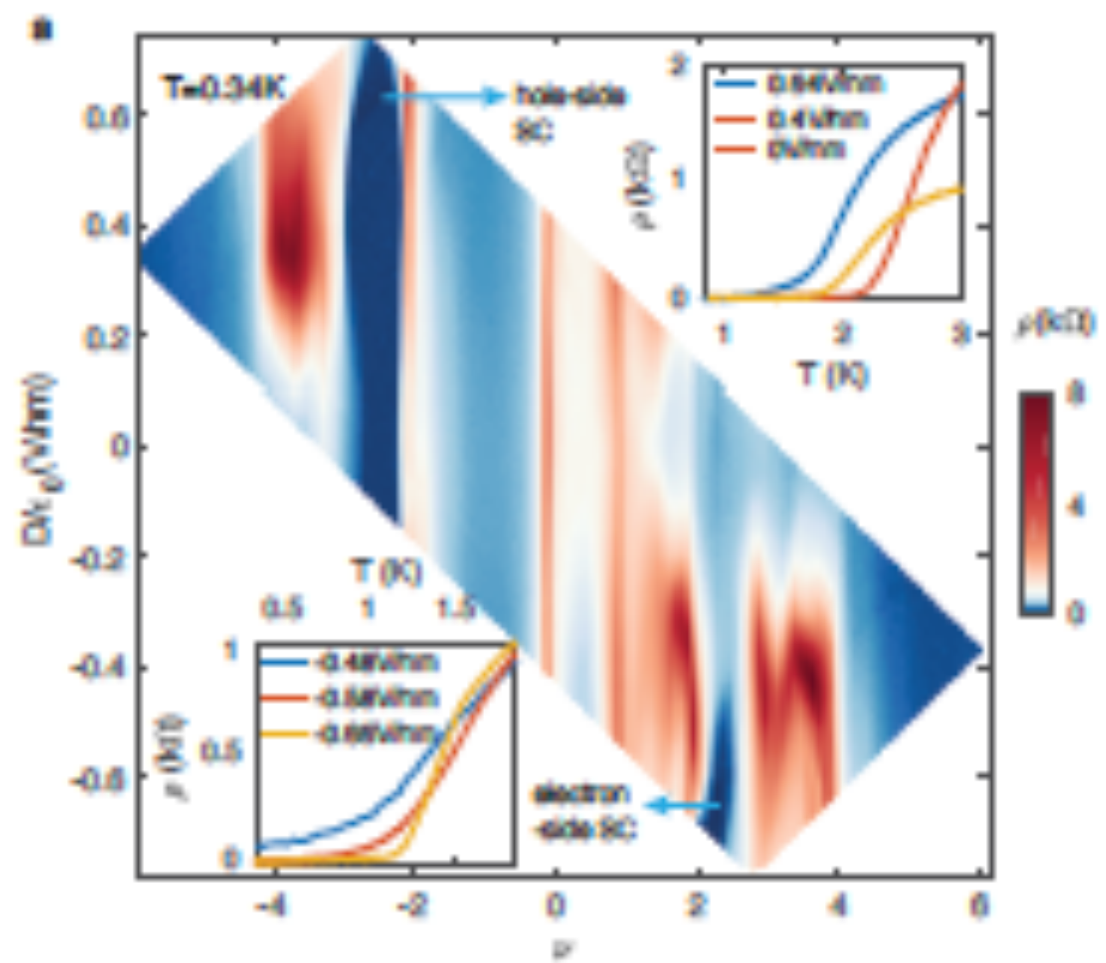
Bernd Matthias (1918-1980)

1. high symmetry is good, cubic symmetry is the best
2. high density of electronic states is good
3. stay away from oxygen
4. stay away from magnetism
5. stay away from insulators
6. stay away from theorists.

Alternating twist trilayer -EXPT

Strong coupling superconductivity
associated with |nu|=2
Tuning by displacement field.





Hao et al.

Park et al.

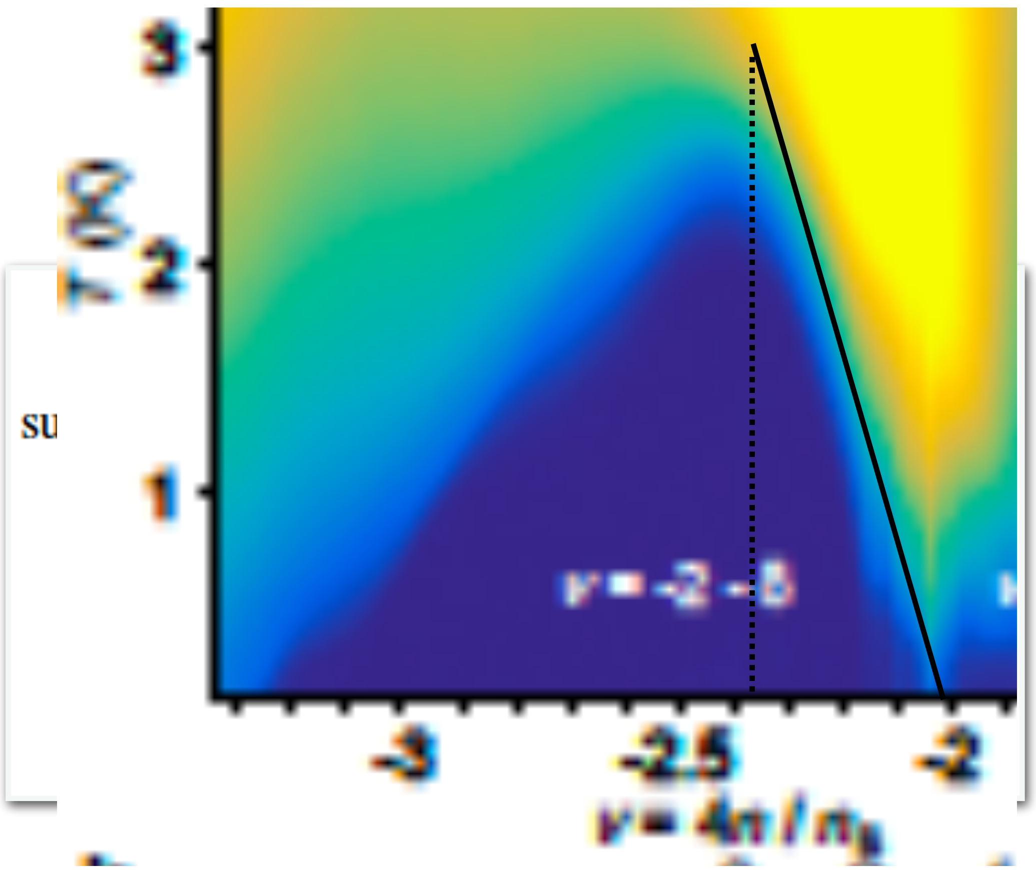
Tunable Phase Boundaries and Ultra-Strong Coupling Superconductivity in Mirror Symmetric Magic-Angle Trilayer Graphene

Jeong Min Park,[1,*] Yuan Cao,[1,*,†] Kenji Watanabe,[2]
Takashi Taniguchi,[2] and Pablo Jarillo-Herrero[1,†]

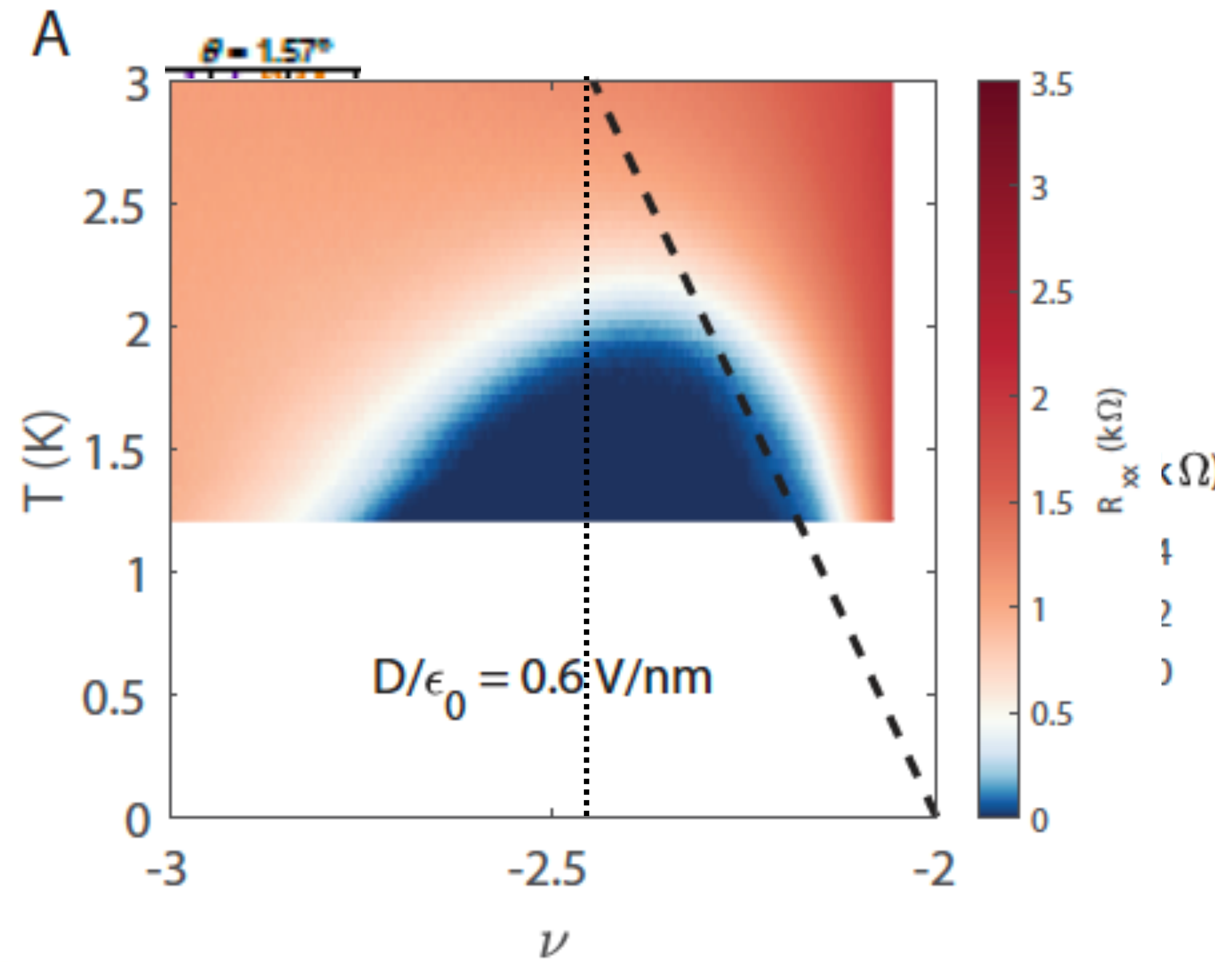
Electric field tunable unconventional superconductivity in alternating twist magic-angle trilayer graphene

Zeyu Hao[1†], A. M. Zimmerman[1†], Patrick Ledwith[1], Eslam Khalaf[1], Danial Haie Najafabadi[1], Kenji Watanabe[2], Takashi Taniguchi[3], Ashvin Vishwanath[1] & Philip Kim[1*]

Experiments on Alternating Twist Trilayer



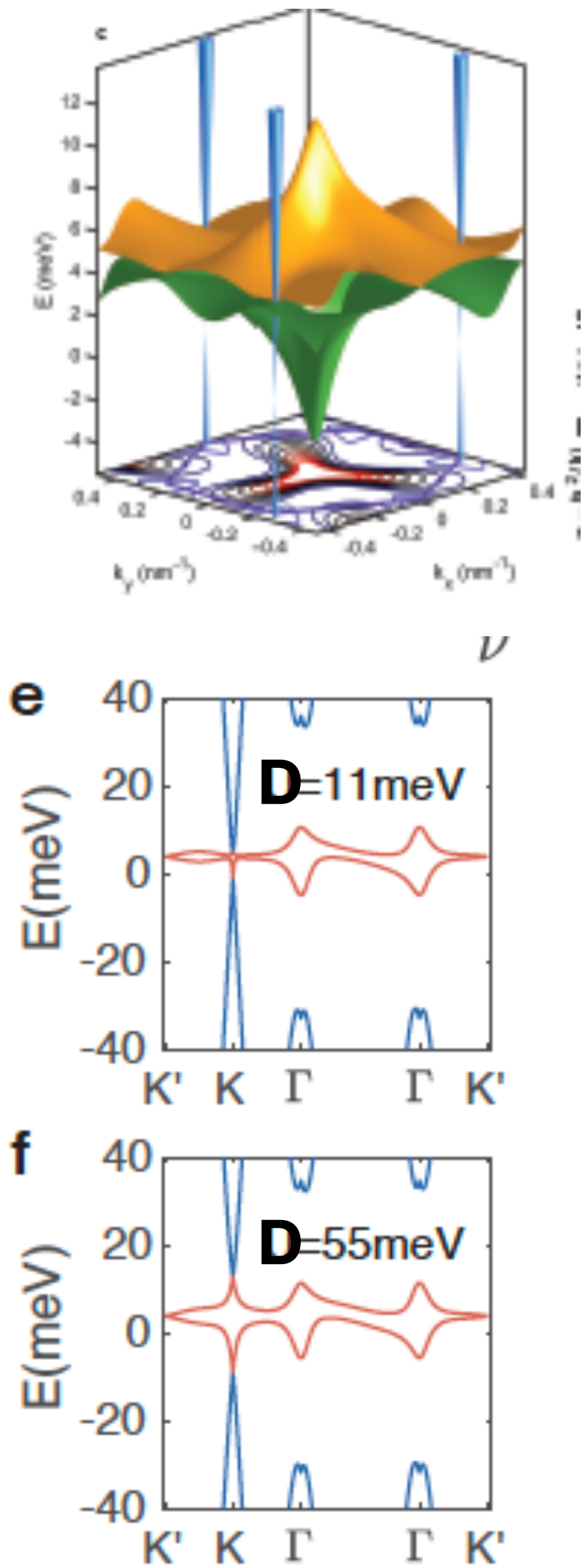
Pablo's talk - Moire 3.0



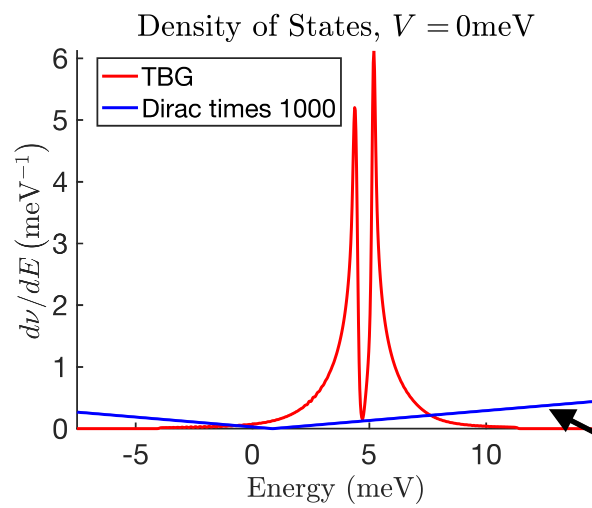
Hao et al.

Role of Dirac fermions?

Alternating twist trilayer



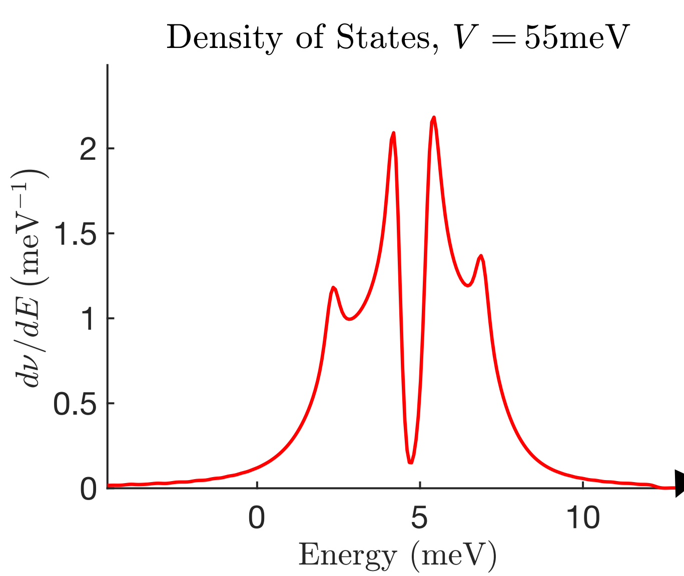
Patrick Ledwith



Mirror symmetry separates Dirac & flat bands at D=0

Very small effect of Dirac dispersion on DOS.

Large velocity



Non-unitary versus unitary pairing

Symmetry constraints on superconductivity in twisted bilayer graphene: Fractional vortices, $4e$ condensates or non-unitary pairing

Eslam Khalaf, Patrick Ledwith, and Ashvin Vishwanath
Department of Physics, Harvard University, Cambridge, MA 02138

arXiv:2012.05915

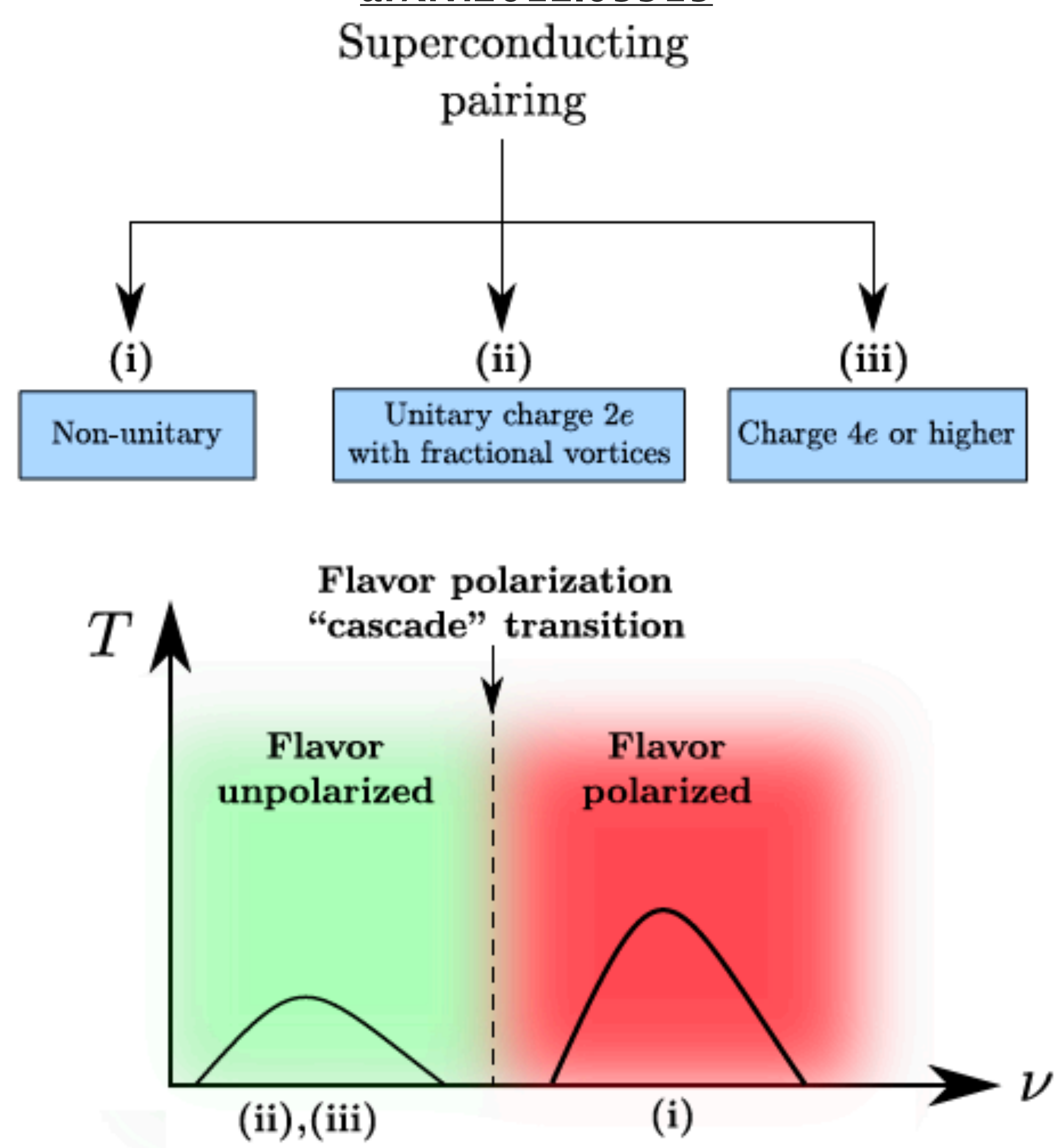
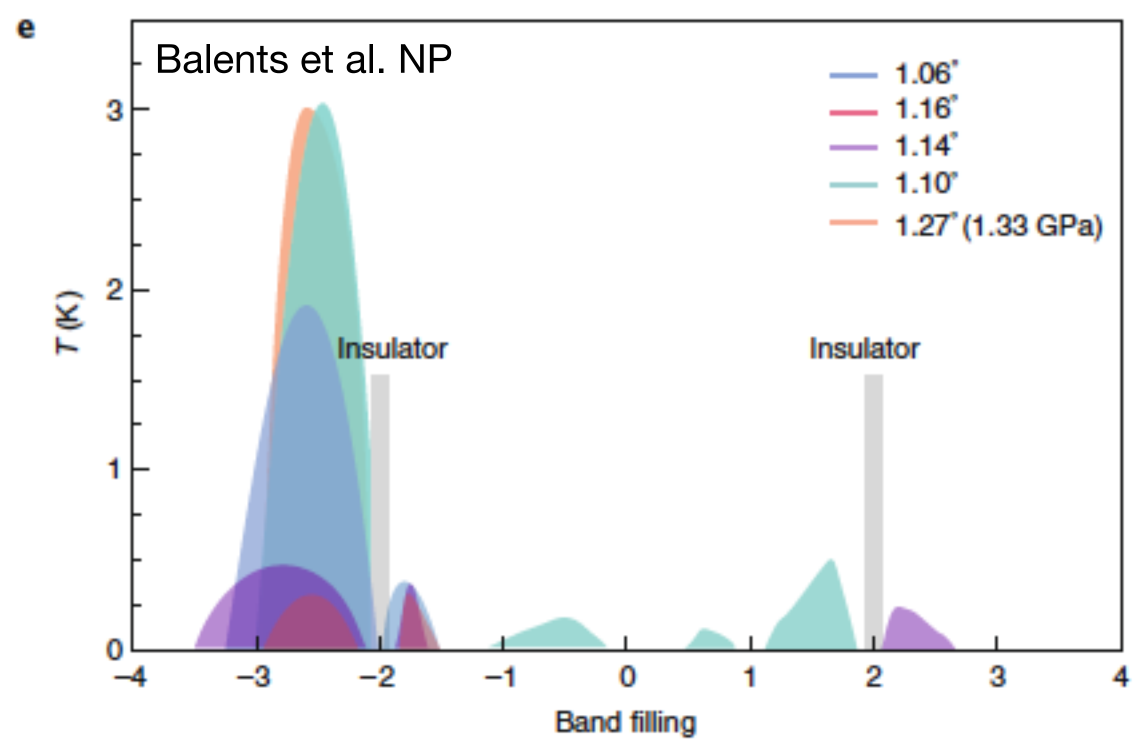
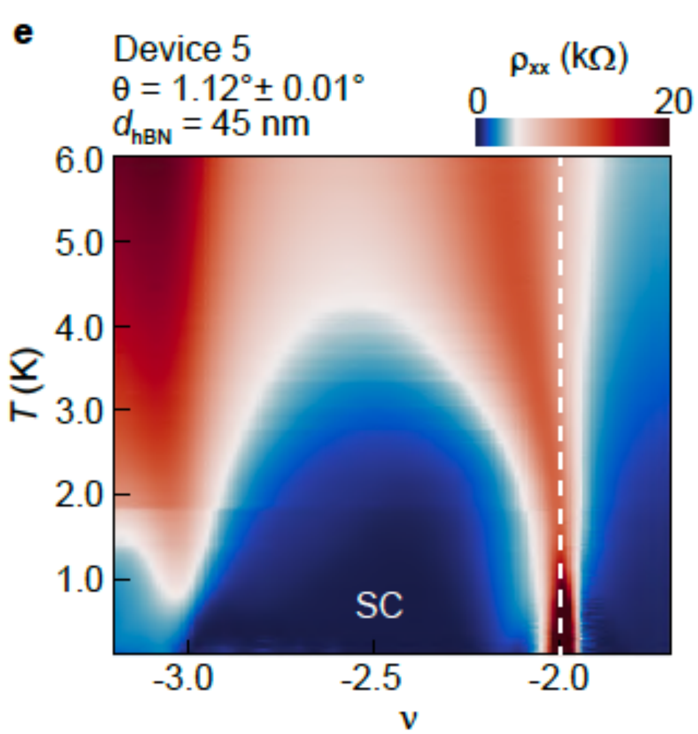


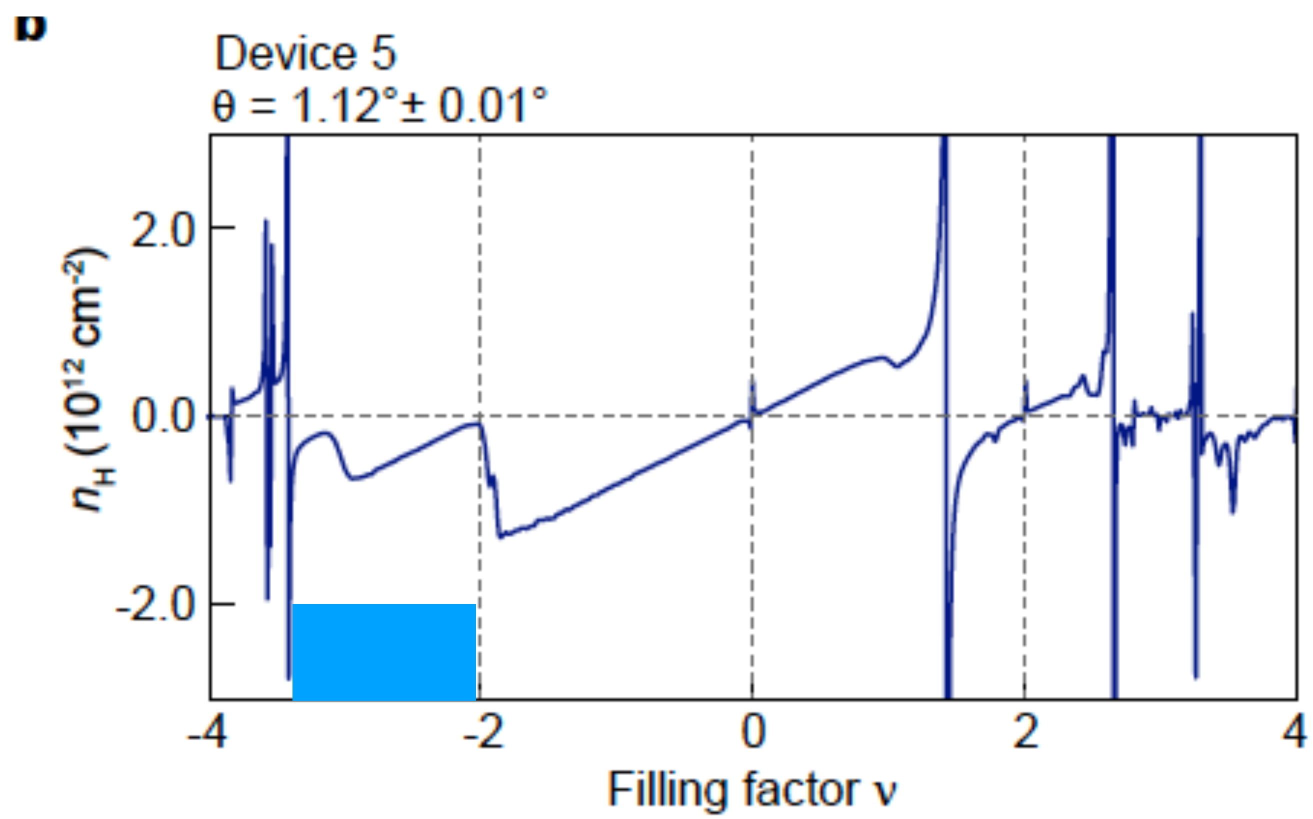
FIG. 1: Schematic illustration of the three possibilities for superconducting pairing with the global symmetry $G = \mathrm{U}(1)_V \times \mathrm{SU}(2)_K \times \mathrm{SU}(2)_{K'}$ or $G_{\mathrm{high}} = \mathrm{SU}(4)$.

Superconductivity without Flavor Reset?





Saito et al. `19

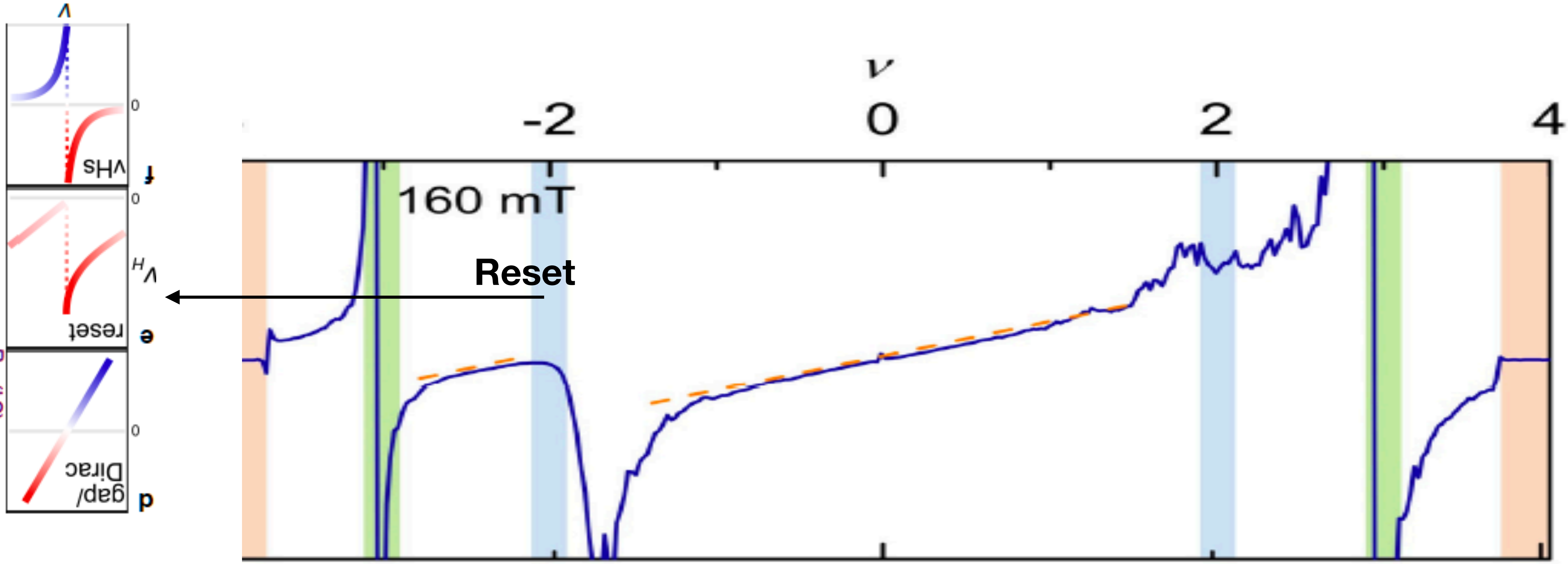


Superconductivity largely tied to nu=-2, +2

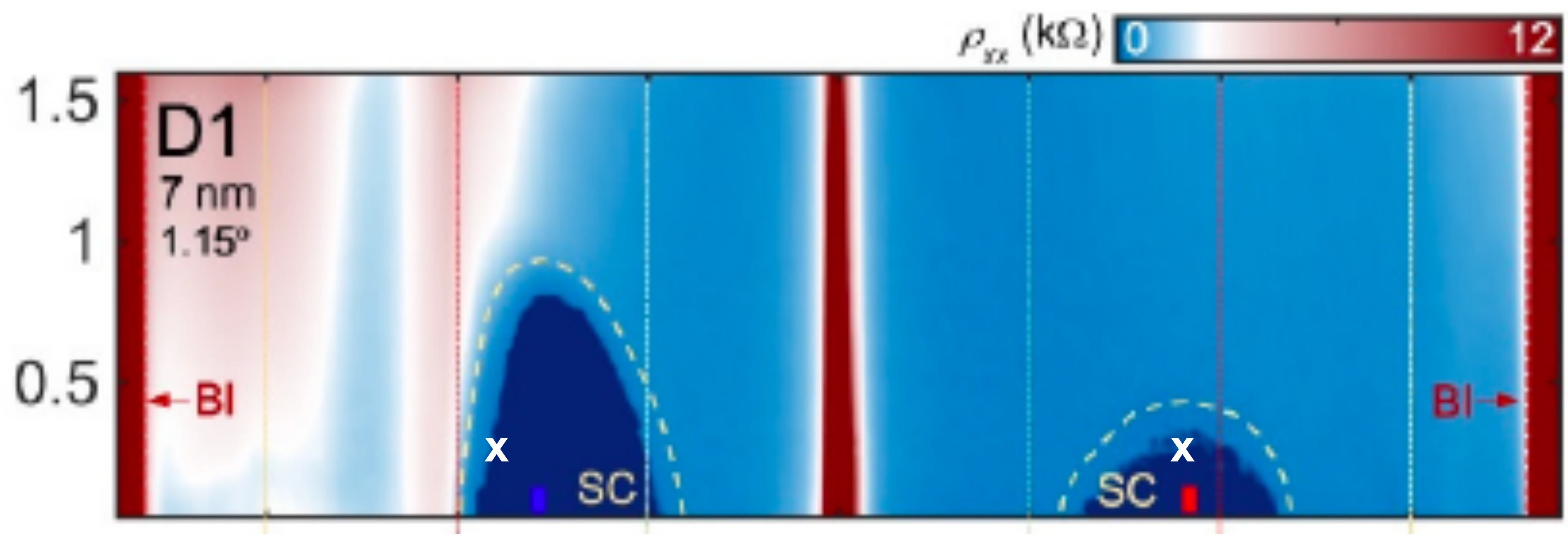
Emerges after `reset’ where two e/Moire are polarized

Strong particle-hole asymmetry.

Interesting Outliers - Fractional Vortices?



Stepanov…
Efetov 19



Conclusions & Future Directions

- Skyrmions gives a natural mechanism for superconductivity in 2-3 layer graphene.
    - A soluble strong coupling theory - not local DOS related.
    - associated with nu=2
    - Connection to C_2T symmetry, accounts for robust Sc platforms.
    - Predicts new platforms.
- Future Directions
    - measure collective modes to find J, ρ_p, λ.. Quantitative relation between normal state and predictions of skyrmions Sc.
    - We want to incorporate fermions into the theory (coexisting fermions + Skx).
    - Connection to weak coupling theories?
    - Distinct superconductors without flavor polarization?

Lecture Notes: **https://scholar.harvard.edu/avishwanath/teaching**

Collaborators

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<u>Theory</u>: Jong Yeon Lee, Daniel Parker, G. Tarnopolsky, A. Kruchkov, Adrian Po, T. Senthil, Liujun Zou

<u>Experiment</u>: Kim group, Yacoby group, Yazdani group.

<u>Discussions</u>: Pablo Jarillo Herrero, Yuan Cao, Jane Park, Andrea Young, Allan MacDonald

Conclusions

DMRG: Model and phase diagram

- iDMRG for coupled Landau level model on a cylinder ($L_y = 8\text{-}12\ \ell_B$)

Ippoliti *et al*, PRB (2018)

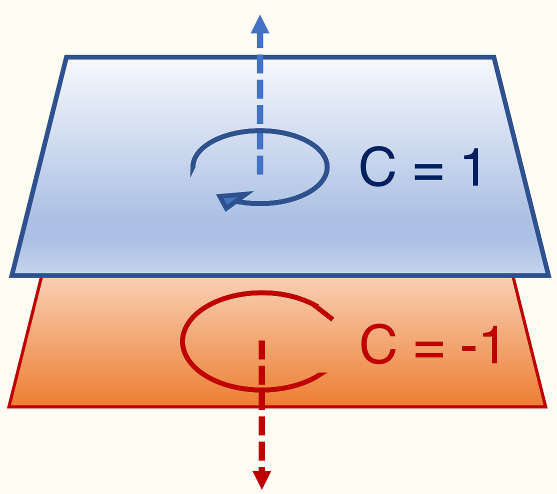
$$H = \psi^\dagger \frac{(\mathbf{p} + e\gamma^z \mathbf{A})^2}{2m}\psi + \frac{1}{2}\int : n(r) V_C(r-r') n(r') : - E_C \ell_B^2 \sum_{i=x,y,z} J_i : \left(\psi^\dagger \gamma^z \eta^i \psi(r)\right)^2 :$$

Kinetic term

Coulomb repulsion

AF super-exchange

γ = layer, η = spin



$$J_x = J_y = J + \lambda, J_z = J - \lambda$$

Easy plane/easy axis anisotropy

Isotropic super-exchange

Purely repulsive model for J < 3.24 ($d_s = 3\ell_B$)

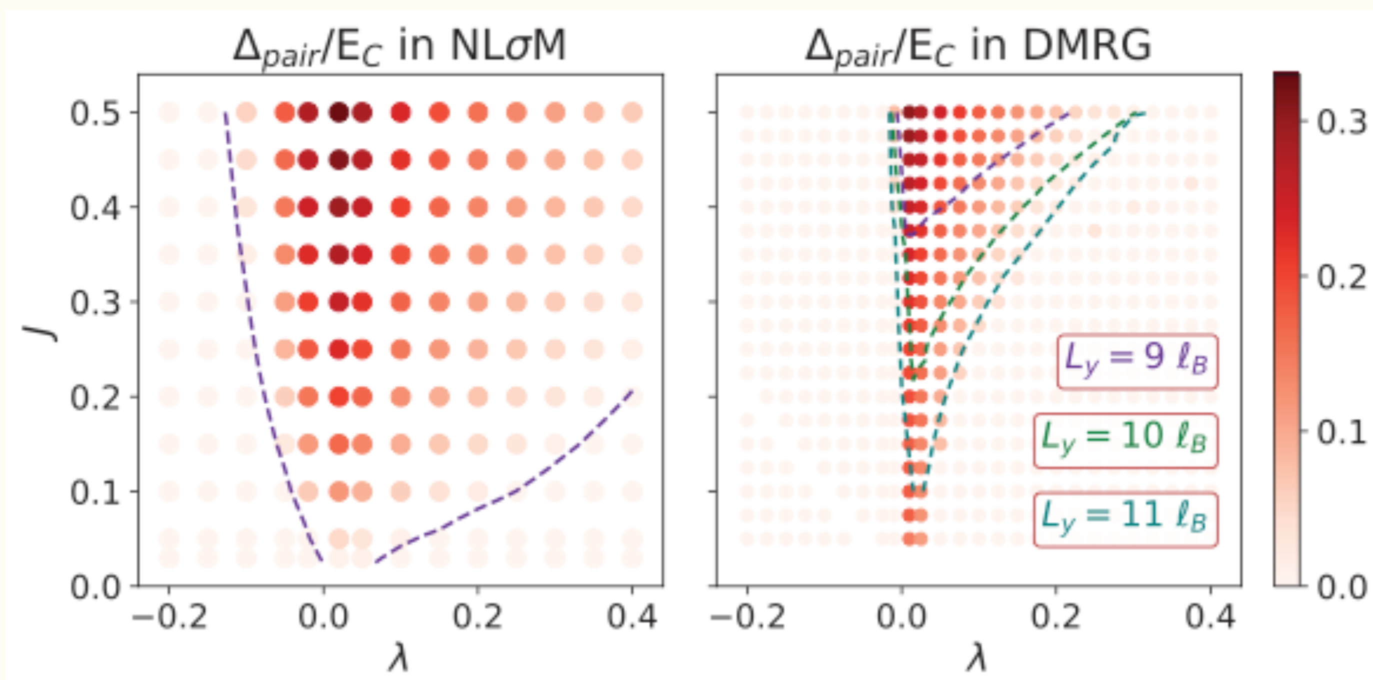
Related work: Kang and Vafek, PRB (2020)
Soejima, Parker *et al*, PRB (2020)
Eugenio and Dag, arXiv: 2004.10363

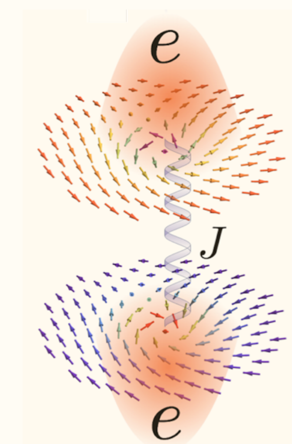
S. Chatterjee, M. Ippoliti, M. P. Zaletel, arXiv:2010:01144

Evidence for skyrmion-pairing

- NLSM + Segment DMRG to determine energy of charged excitations

$$\boxed{\Delta_{pair} = 2\,E_{1e} - E_{2e}}$$



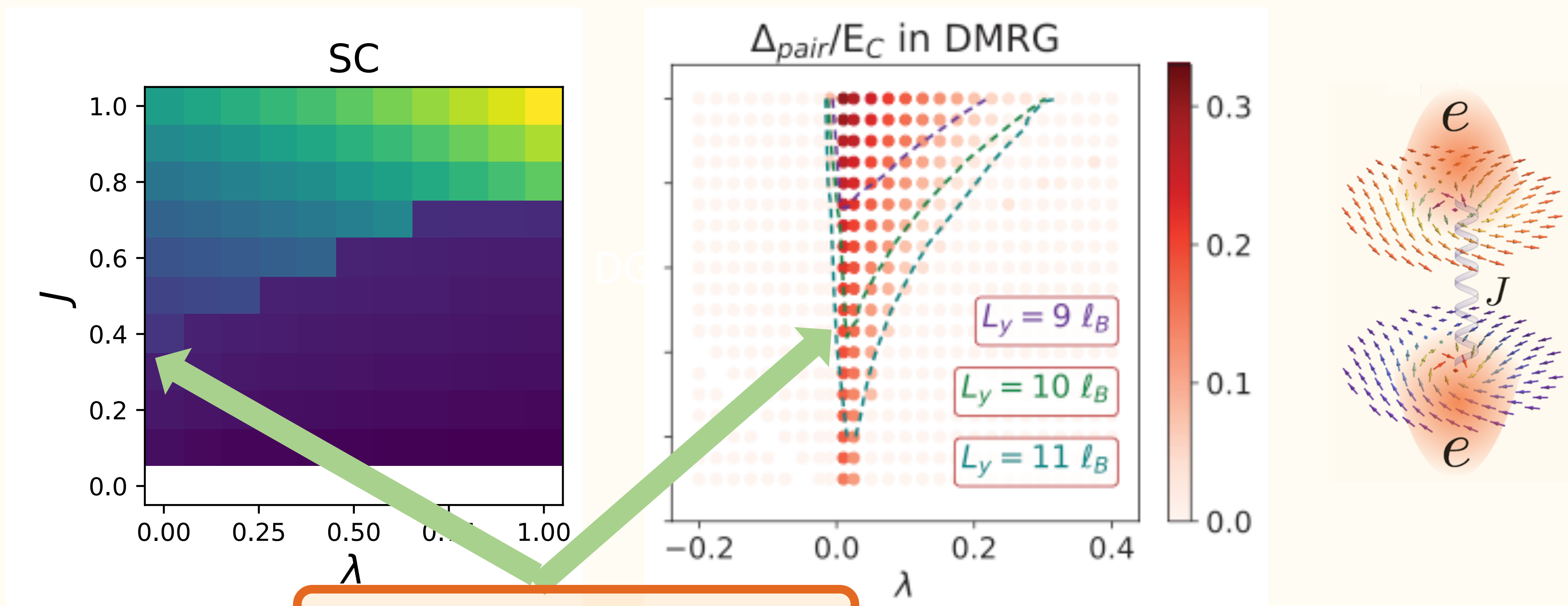


- Numerics for quantum system confirm classical expectations!

S. Chatterjee, M. Ippoliti, M. P. Zaletel, arXiv:2010:01144

Evidence for skyrmion-pairing

- Critical $J_*(\lambda) \to 0$ as $\lambda \to 0$, indicative of collective pairing mechanism
- Pairing is much more favorable in the easy plane case (good for MAG!)



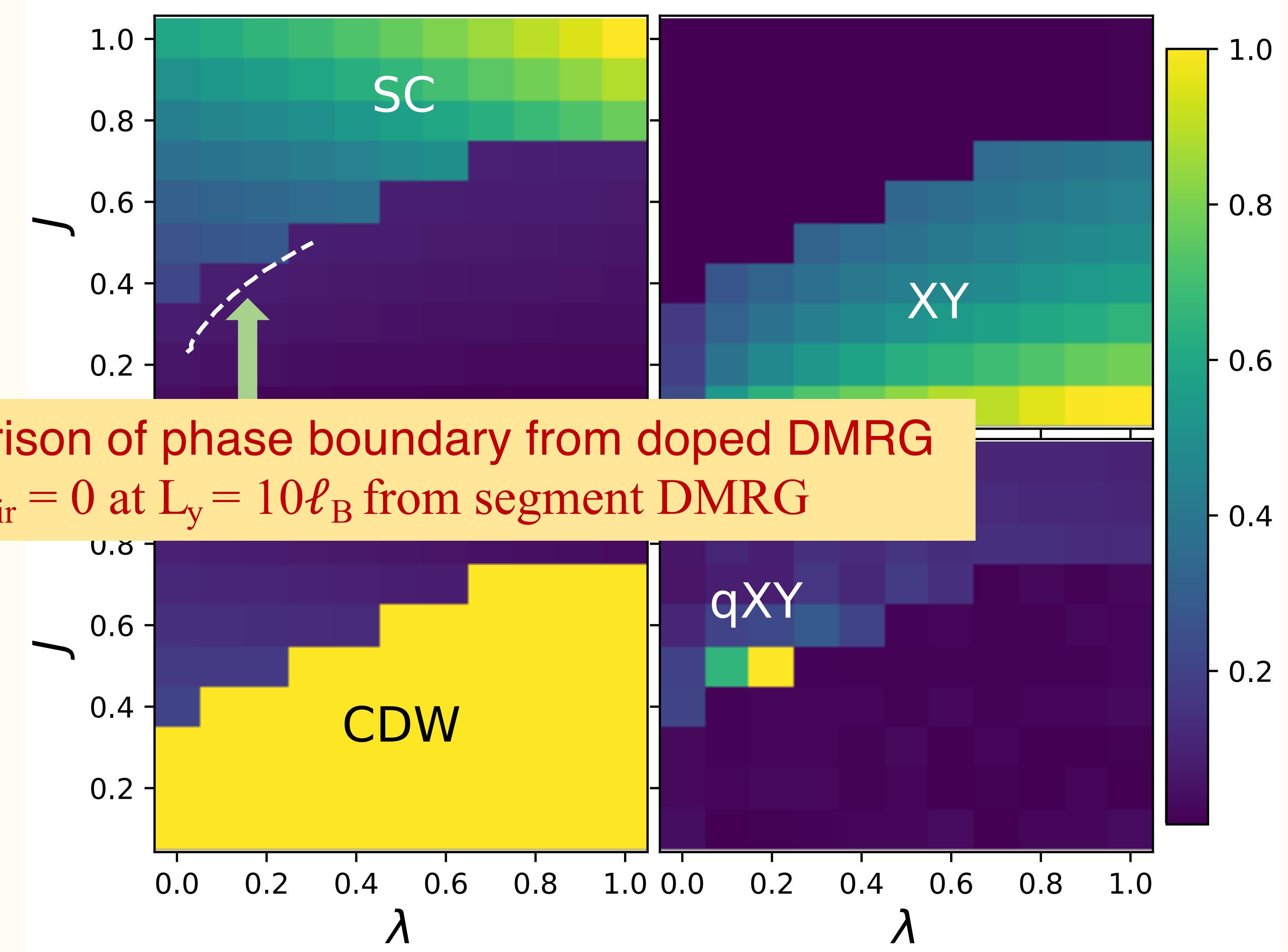
$J_*(\lambda{=}0, L_y = 10\ell_B) \cong 0.3$

- Good qualitative agreement between quantum and classical numerics

S. Chatterjee, M. Ippoliti, M. P. Zaletel, arXiv:2010:01144

Evidence for skyrmion-pairing

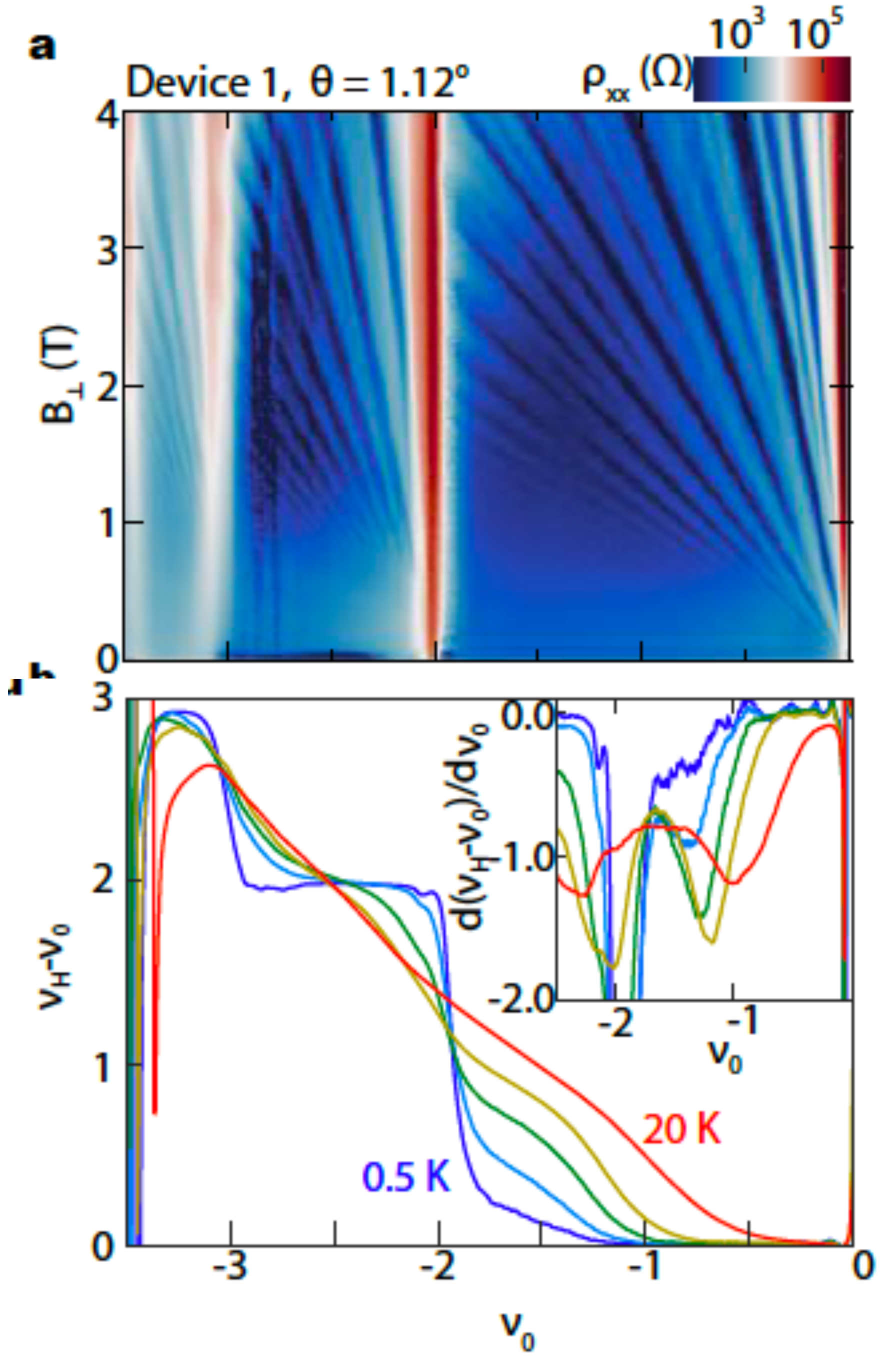
Phase diagram at doping 2 + 1/4

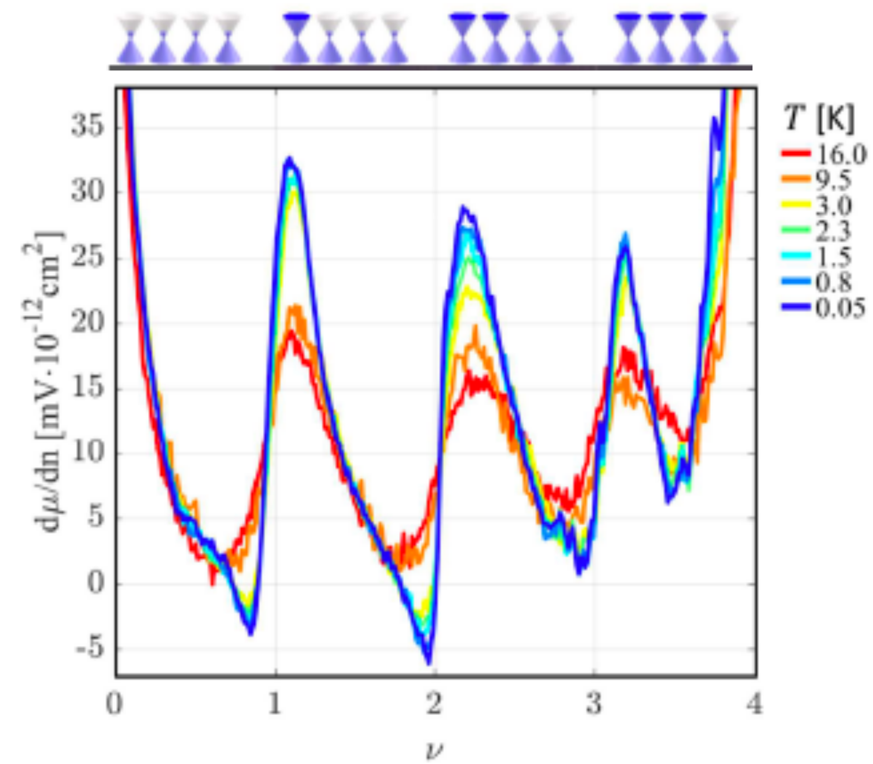


Comparison of phase boundary from doped DMRG with $\Delta_{pair} = 0$ at $L_y = 10\ell_B$ from segment DMRG

S. Chatterjee, M. Ippoliti, M. P. Zaletel, arXiv:2010:01144

Flavor Polarization

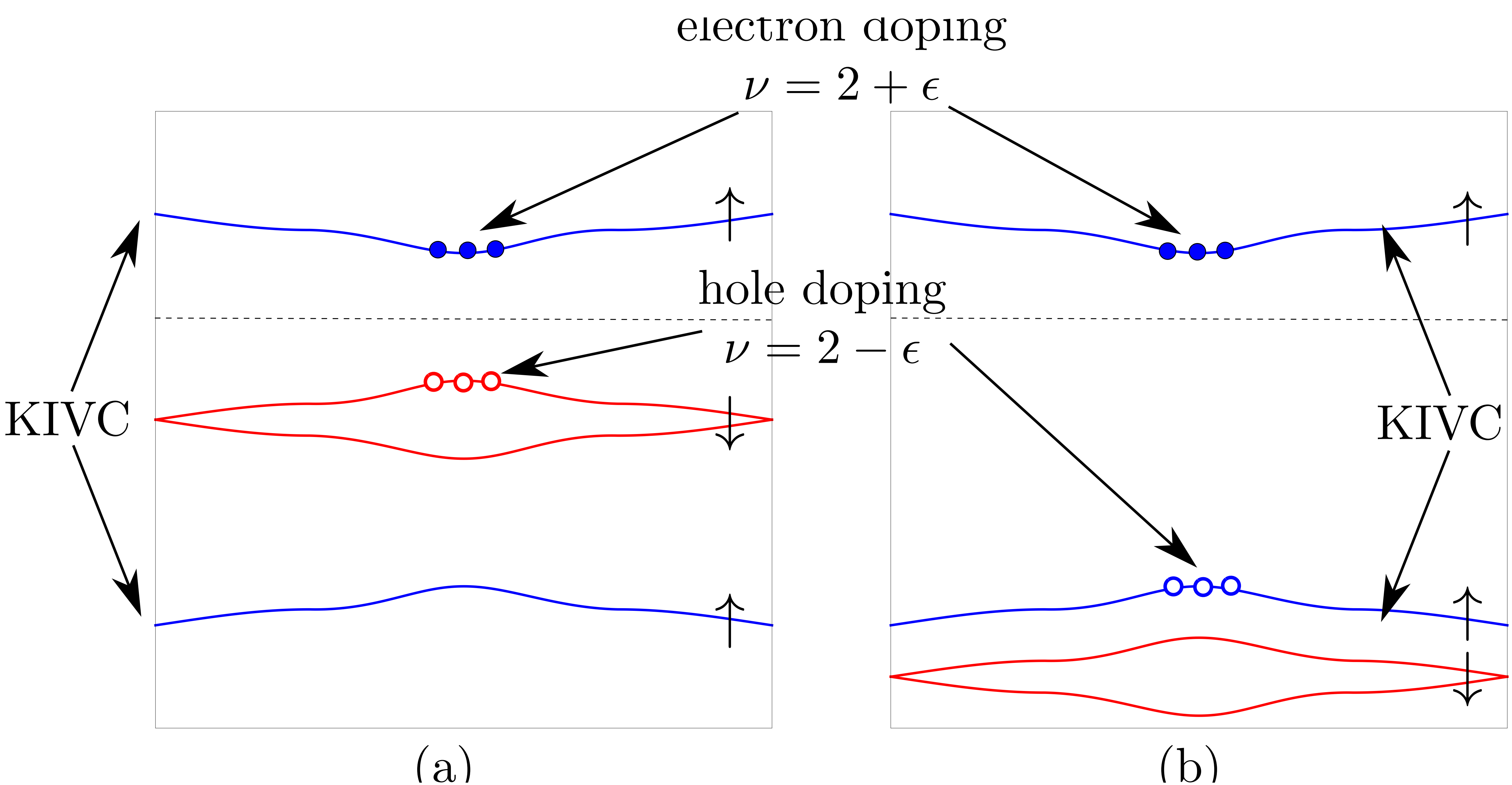




Zondiner et al. arXiv:1912.06150
Wong et al. arxiv: 1912.06145

Saito et al.arXiv:2008.10830

Including Spin



KIVC with spin

See also: Christos…Sachdev (2020)

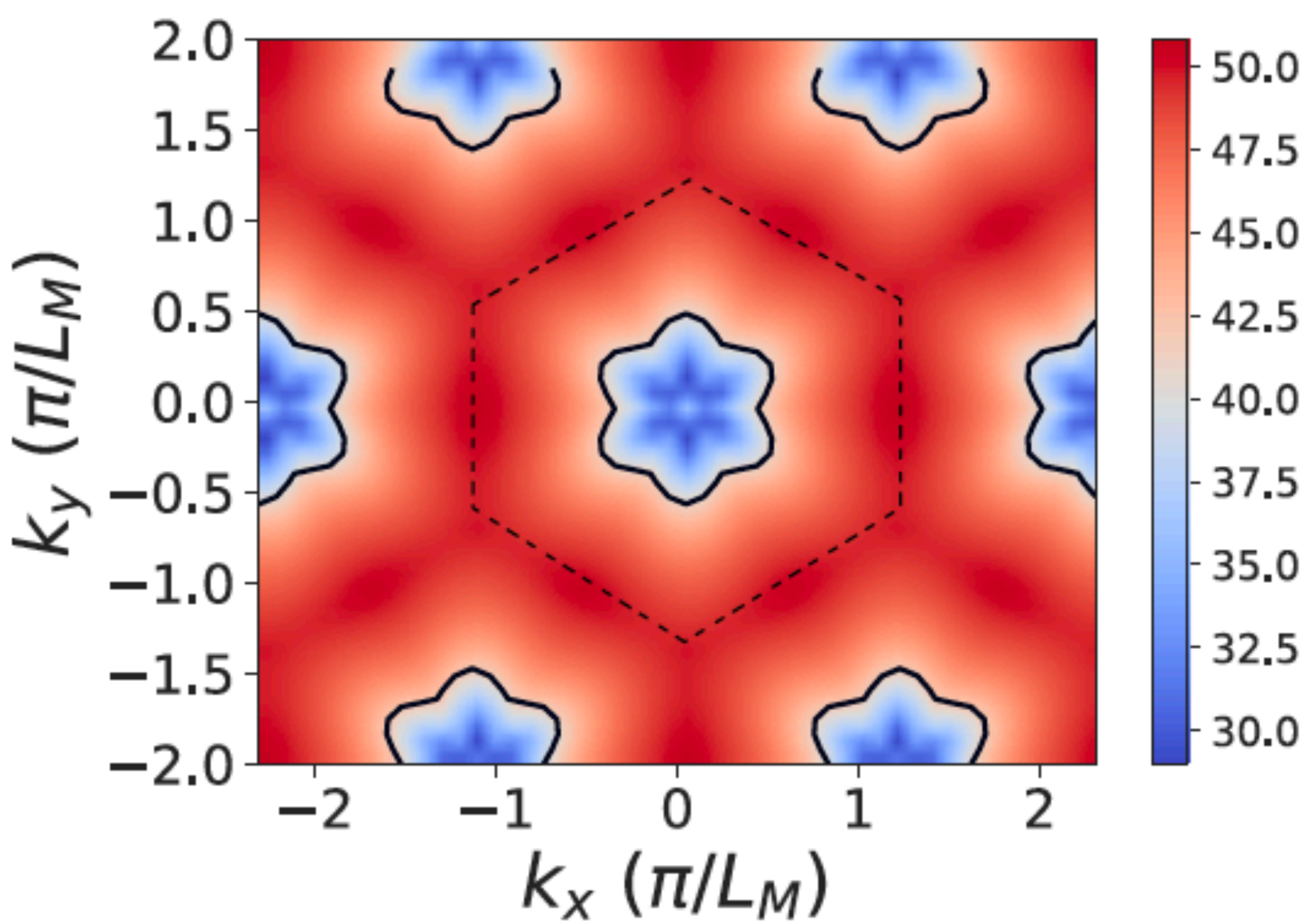


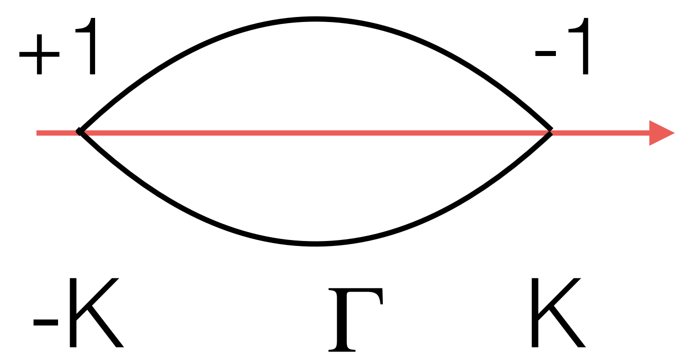
FIG. 7: Energy contour (in meV) for pair energy $= \Delta_{\mathrm{KIVC}}(\boldsymbol{k})$ in the moiré BZ for $d_s = 20$ nm, $\theta = 1.05°$ (repeated zone scheme). Beyond $\nu_c \approx 0.2$, doped charges go in as $2e$ skyrmions, within the estimates of Section III C.

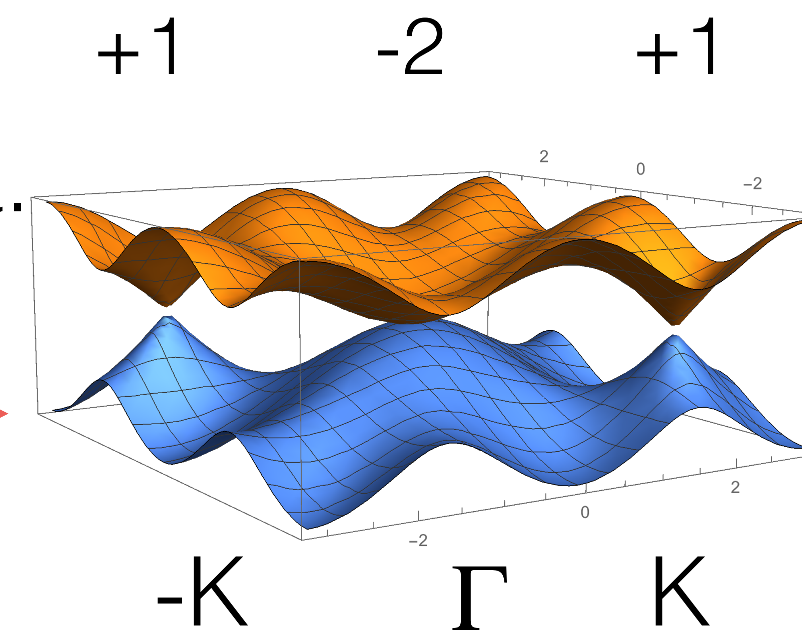
Topological Obstruction to 2 band model

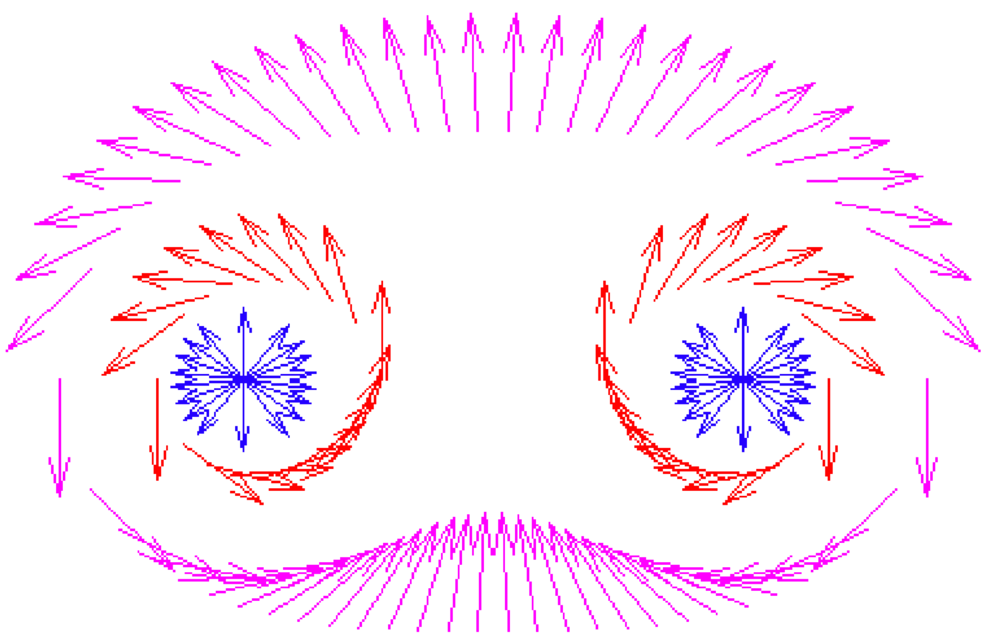
However, the two Dirac points have the **same** chirality.

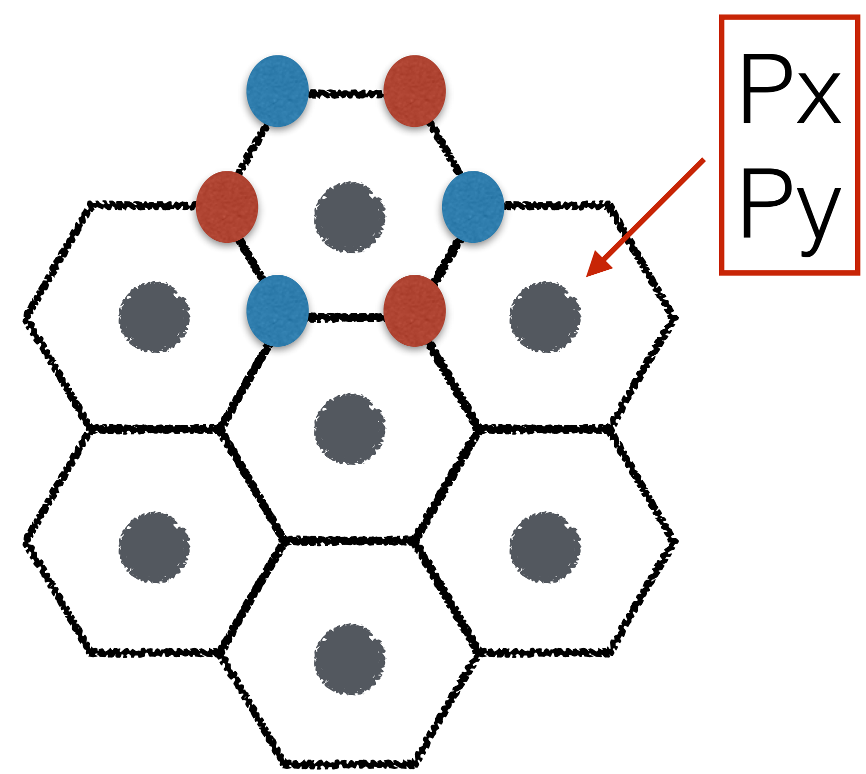
Origin - Dirac dispersion

Any tight binding model - opposite/extra.





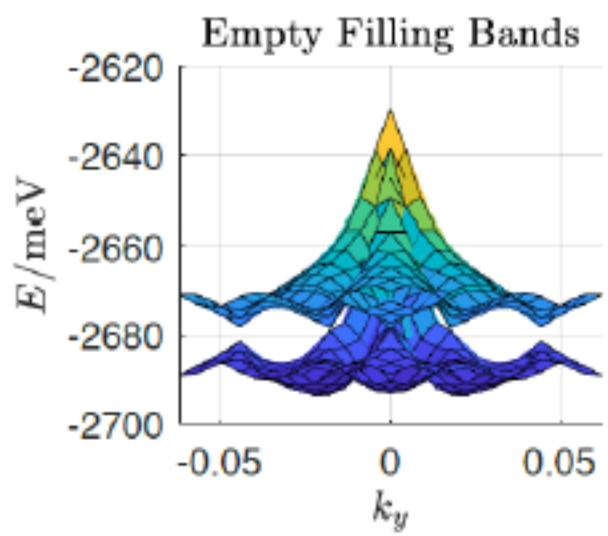


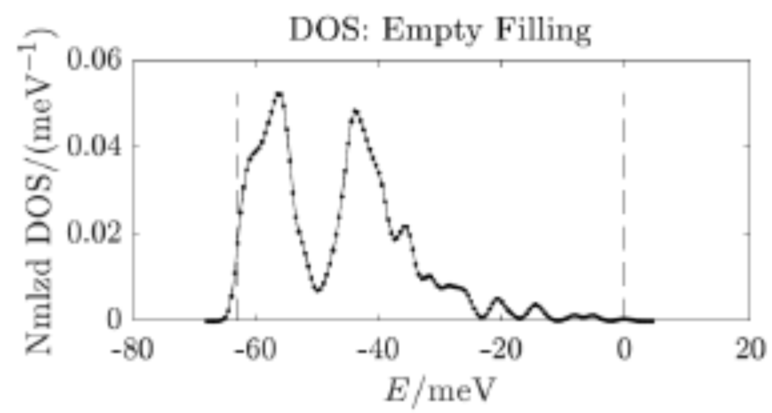


[Castro Neto et al., PRB 2011, Goerbig & Montambaux, 2017, …]

Dispersion

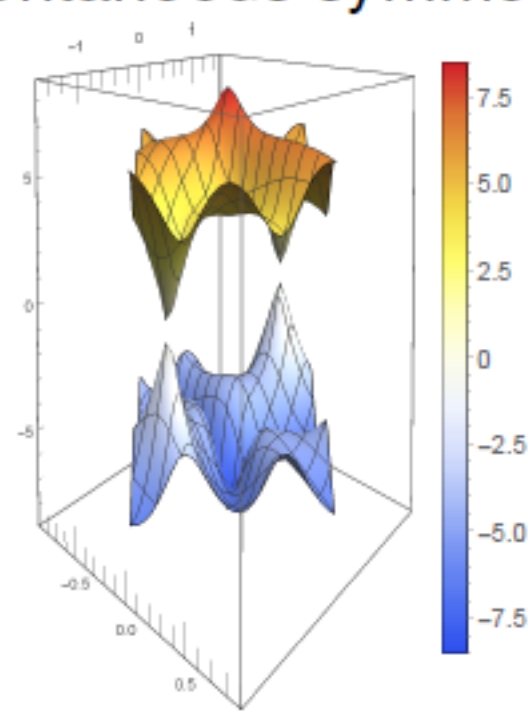
- For $w_0 \neq 0$, bare bandwidth is small at the magic angle
- Renormalization due to interactions $\rightarrow$ bandwidth $\sim$ 10-15 meV
  - Empty filling (Liu, EK, Lee, Vishwanath 19)





Good metric of eventual bandwidth - distance between peaks at empty filling ~10meV

  - Charge neutrality (no spontaneous symmetry breaking)



$\tilde{h} \sim 10meV$ bandwidth

Renormalized Dispersion

From:
Bultnick et al. arXiv:1911.02045

$$P_{\alpha\beta}(k) = \langle c^\dagger_\alpha(k) c_\beta(k)\rangle \ \& \ P = \frac{1}{2}[1+Q]$$

at neutrality

$$\mathcal{H}_{\text{int}} = \sum_{\boldsymbol{k}} c^\dagger_{\boldsymbol{k}} \tilde{h}(\boldsymbol{k}) c_{\boldsymbol{k}} + \frac{1}{2A}\sum_{\boldsymbol{q}} V_{\boldsymbol{q}} \delta\rho_{\boldsymbol{q}} \delta\rho^\dagger_{-\boldsymbol{q}} + \text{const.} \tag{S39}$$

$$\tilde{h}(\boldsymbol{k}) = h_{\nu=0}(\boldsymbol{k}) + \frac{1}{2A}\sum_{\boldsymbol{q}} V_{\boldsymbol{q}} \Lambda_{\boldsymbol{q}}(\boldsymbol{k}) Q_0(\boldsymbol{k}+\boldsymbol{q}) \Lambda^\dagger_{\boldsymbol{q}}(\boldsymbol{k}), \qquad \delta\rho_{\boldsymbol{q}} = \rho_{\boldsymbol{q}} - \bar{\rho}_{\boldsymbol{q}}, \qquad \bar{\rho}_{\boldsymbol{q}} = \frac{1}{2}\sum_{\boldsymbol{G},\boldsymbol{k}} \delta_{\boldsymbol{G},\boldsymbol{q}} \operatorname{tr} \Lambda_{\boldsymbol{G}}(\boldsymbol{k}) \tag{S40}$$

This is the dispersion at neutrality - bigger than BM but similar form since dominates bandwidth

$$Q(k) = \sigma_x e^{i\phi_0(k)\sigma_z\tau_z}$$

$$h(\boldsymbol{k}) = h_{\text{BM}}(\boldsymbol{k}) - \frac{1}{2A}\sum_{\boldsymbol{G}} V_{\boldsymbol{G}} \Lambda_{\boldsymbol{G}}(\boldsymbol{k}) \sum_{\boldsymbol{k}'} \operatorname{tr} \Lambda^*_{\boldsymbol{G}}(\boldsymbol{k}') + \frac{1}{2A}\sum_{\boldsymbol{q}} V_{\boldsymbol{q}} \Lambda_{\boldsymbol{q}}(\boldsymbol{k}) \Lambda^\dagger_{\boldsymbol{q}}(\boldsymbol{k}) + \frac{1}{2A}\sum_{\boldsymbol{q}} V_{\boldsymbol{q}} \Lambda_{\boldsymbol{q}}(\boldsymbol{k}) Q_0(\boldsymbol{k}+\boldsymbol{q}) \Lambda^\dagger_{\boldsymbol{q}}(\boldsymbol{k}) \tag{S38}$$

Rearranging

$$h(\boldsymbol{k}) = \underset{\uparrow \atop h_{\text{BM}}}{h_{\nu=0}(\boldsymbol{k})} - \frac{1}{A}\sum_{\boldsymbol{G}} V_{\boldsymbol{G}} \Lambda_{\boldsymbol{G}}(\boldsymbol{k}) \sum_{\boldsymbol{k}'} \operatorname{tr} P_0(\boldsymbol{k}') \Lambda^*_{\boldsymbol{G}}(\boldsymbol{k}') + \frac{1}{A}\sum_{\boldsymbol{q}} V_{\boldsymbol{q}} \Lambda_{\boldsymbol{q}}(\boldsymbol{k}) P^T_0(\boldsymbol{k}+\boldsymbol{q}) \Lambda^\dagger_{\boldsymbol{q}}(\boldsymbol{k}) \tag{S35}$$

This is the dispersion measured in STM at nu=-4

Symmetries of the BM model

- Spatial symmetries (σ sublattice, τ valley, μ layer, s spin)

$$C_3 f_{\boldsymbol{k}} C_3^{-1} = e^{-\frac{2\pi}{3} i\tau_z\sigma_z}, \qquad C_2 f_{\boldsymbol{k}} C_2^{-1} = \sigma_x\tau_x f_{-\boldsymbol{k}}, \qquad M_y f_{\boldsymbol{k}} M_y^{-1} = \sigma_x\mu_x f_{m_y\boldsymbol{k}}$$

- Time-reversal symmetry
- Decoupled valleys + no spin-orbit coupling

$$\mathcal{T} f_{\boldsymbol{k}} \mathcal{T}^{-1} = \tau_x f_{-\boldsymbol{k}}, \qquad \mathcal{T} i \mathcal{T}^{-1} = -i$$

$$\implies \mathrm{U}(2)_K \times \mathrm{U}(2)_{K'} \equiv \mathrm{U}_C(1)\times\mathrm{U}_V(1)\times\mathrm{SU}(2)_K\times\mathrm{SU}(2)_{K'}, \quad \text{generators} : \tau_{0,z}s_{0,x,y,z}$$

- Small angle $\boldsymbol{\sigma}_\theta \approx \boldsymbol{\sigma} \implies$ Particle-hole symmetry $\mathcal{P} f_{\boldsymbol{k}} \mathcal{P}^{-1} = i\sigma_x\mu_y f^\dagger_{-\boldsymbol{k}}$
- Time-reversal + Particle-hole → Chiral symmetry

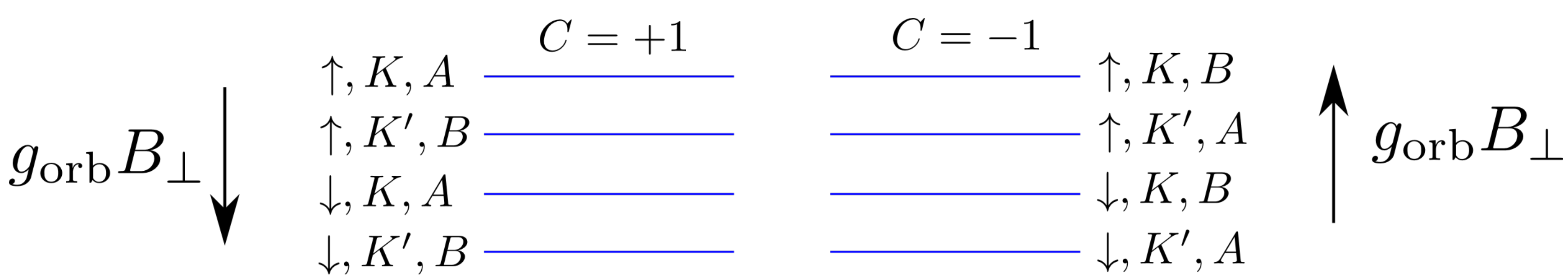
$$\mathcal{C} = i\mathcal{P}\mathcal{T} \implies \mathcal{C} f_{\boldsymbol{k}} \mathcal{C}^{-1} = \tau_x\sigma_x\mu_y f^\dagger_{\boldsymbol{k}}, \quad \mathcal{C} i \mathcal{C}^{-1} = -i$$

- $\mathcal{C}$ flips valley, layer and sublattice.

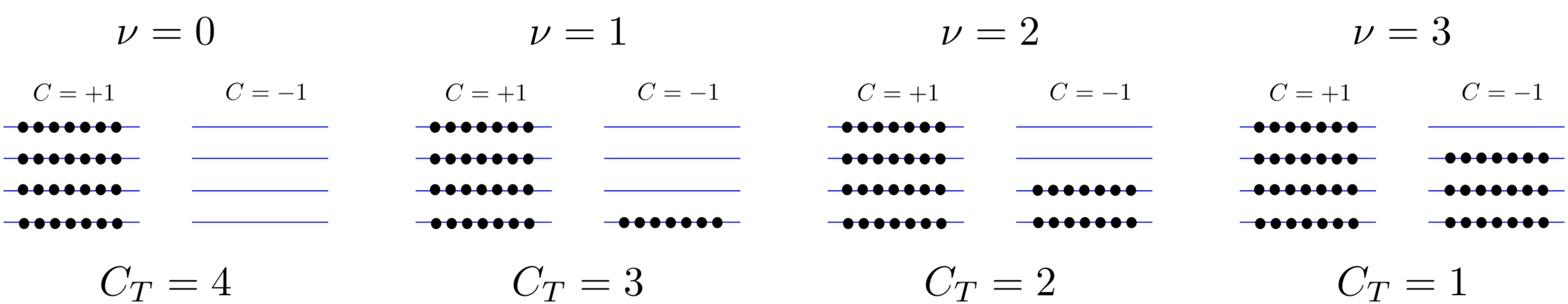
Po *et al*. 2018, Song *et al*. 2018, Yuan, Fu 2018

Chern insulators

- Out-of-plane field: orbital Zeeman effect



- Chern insulators with $|C| = 4 - |\nu|$



Bultinck*, Khalaf*, Liu, Chatterjee, AV, Zaletel PRX 2020