Flavor Magnetism and Superconductivity.

Theory of entwined orders in Magic Angle Graphene (s)

Ashvin Vishwanath

Lecture Notes for More Technical Details:

https://scholar.harvard.edu/avishwanath/teaching

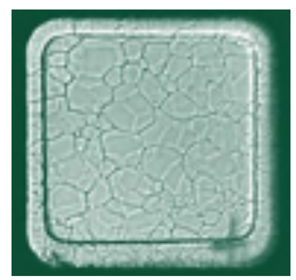
Quantum Phases of Matter

- Landau Orders

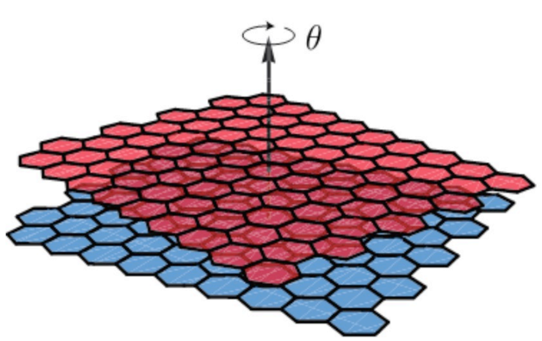
Ferromagnet

Superfluid

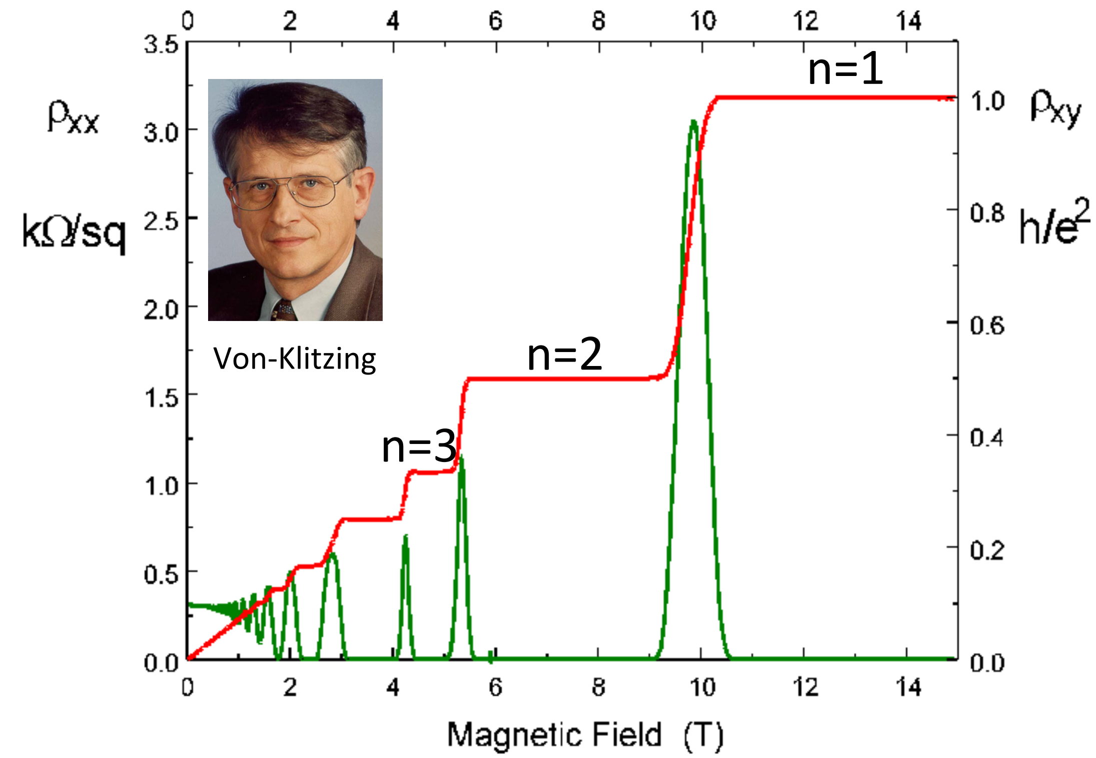
Solid



In many cases the distinguishing feature:
Spontaneous Symmetry Breaking => Landau order parameter.



- topological Phases



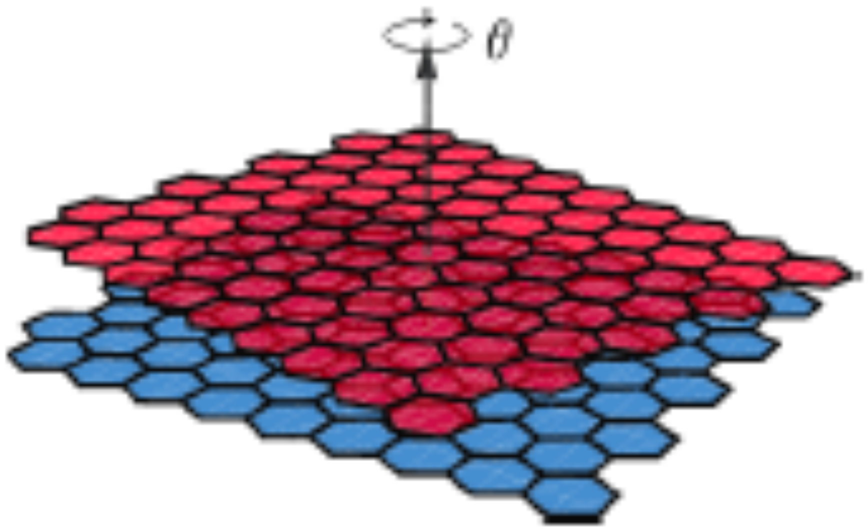
Integer Quantum Hall States

- Different Integers `n' - different phases but same symmetry - topological distinction.

Spontaneous symmetry breaking & topology

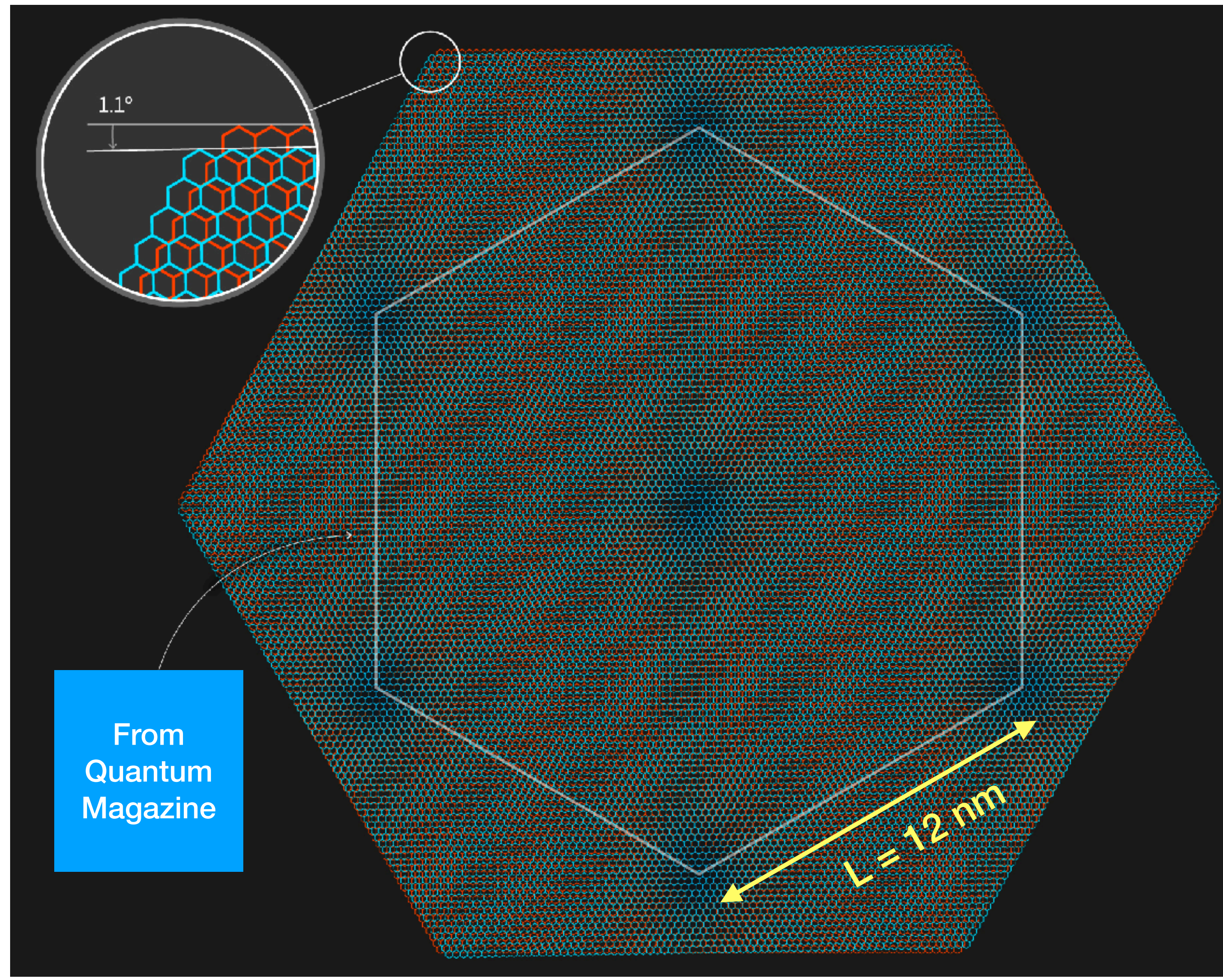
=> Entwined Landau orders

MAGIC ANGLE GRAPHENE



MAGIC ANGLE ~1.1º:
Tunneling time =
Lattice Moire
time

$$\theta \sim 1/60 \text{ radians} \qquad L \sim a/\theta$$

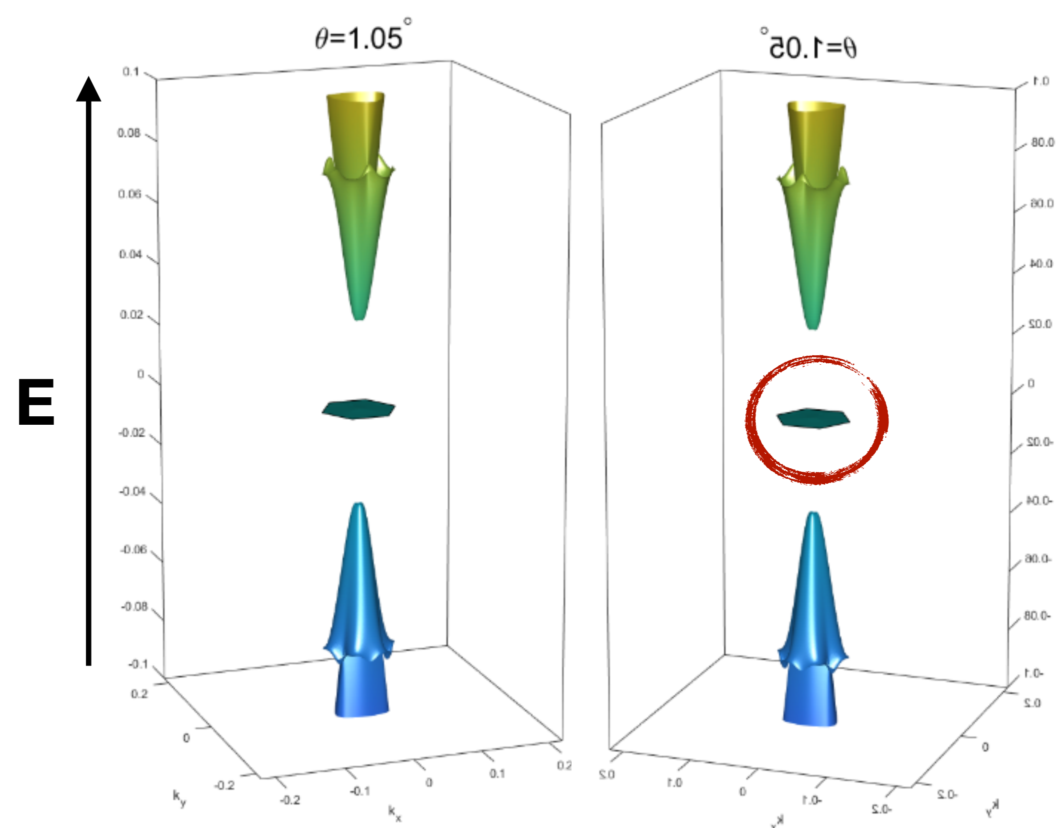


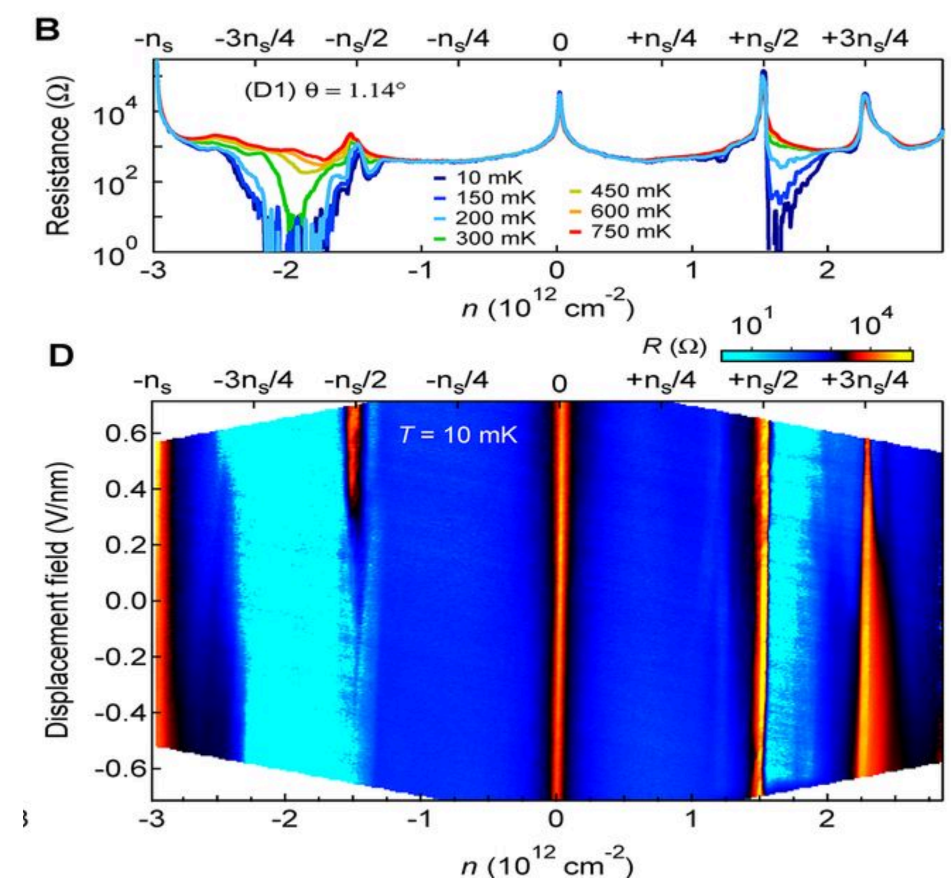
Real **Space**

Experiments

Pablo Jarillo-Herrero’s group (MIT)
Cao *et al.* Nature 556, 80 (2018)
Cao *et al.* Nature 556, 43 (2018)

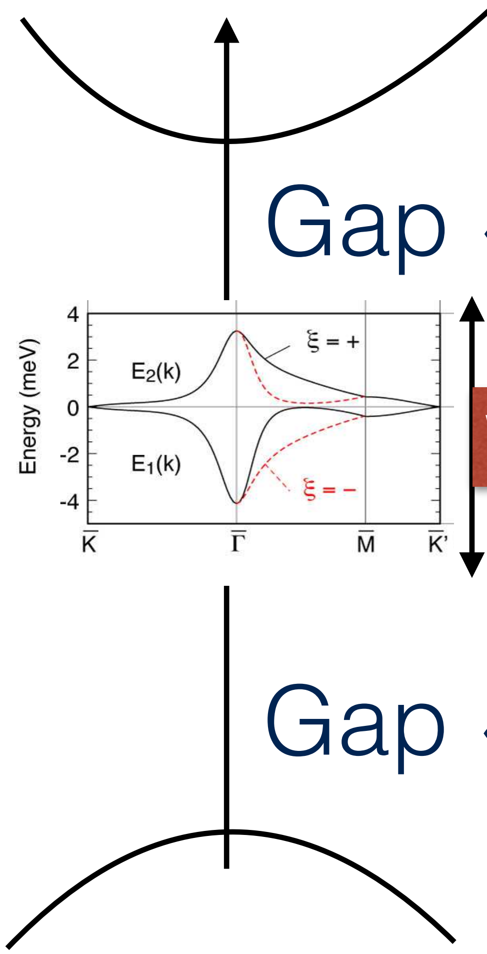
Valley K **Valley K’**





Yankowitz et al. Science

Correlation Effects in Twisted Bilayer Graphene

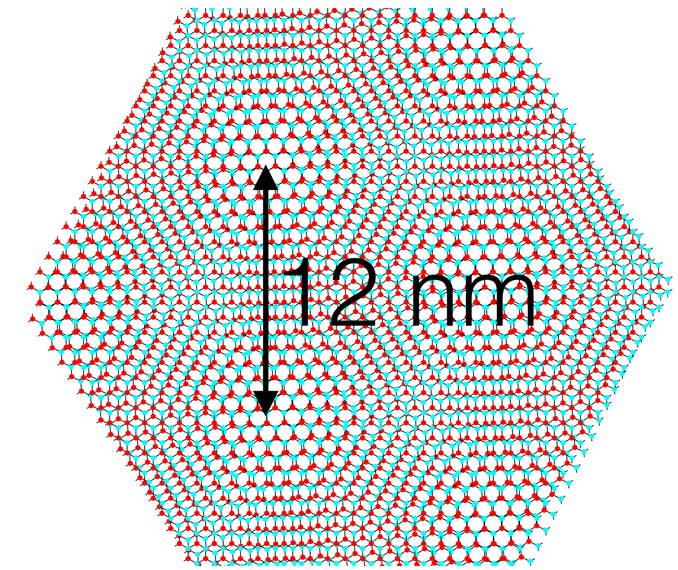


Gap ~ 30 meV

W_{bare} ~ 8 meV

Koshino et al

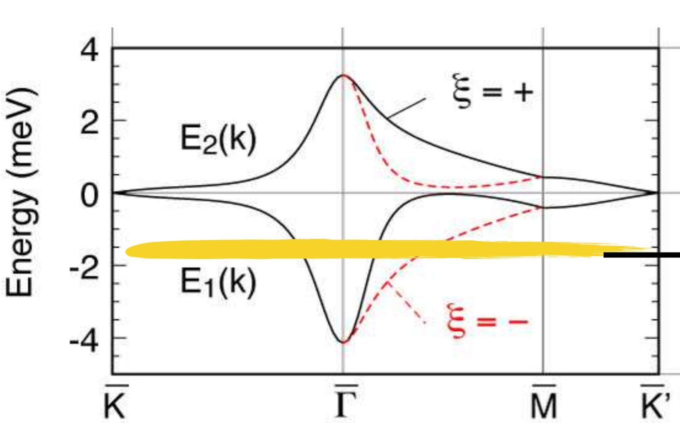
Gap ~ 30 meV



V~30meV

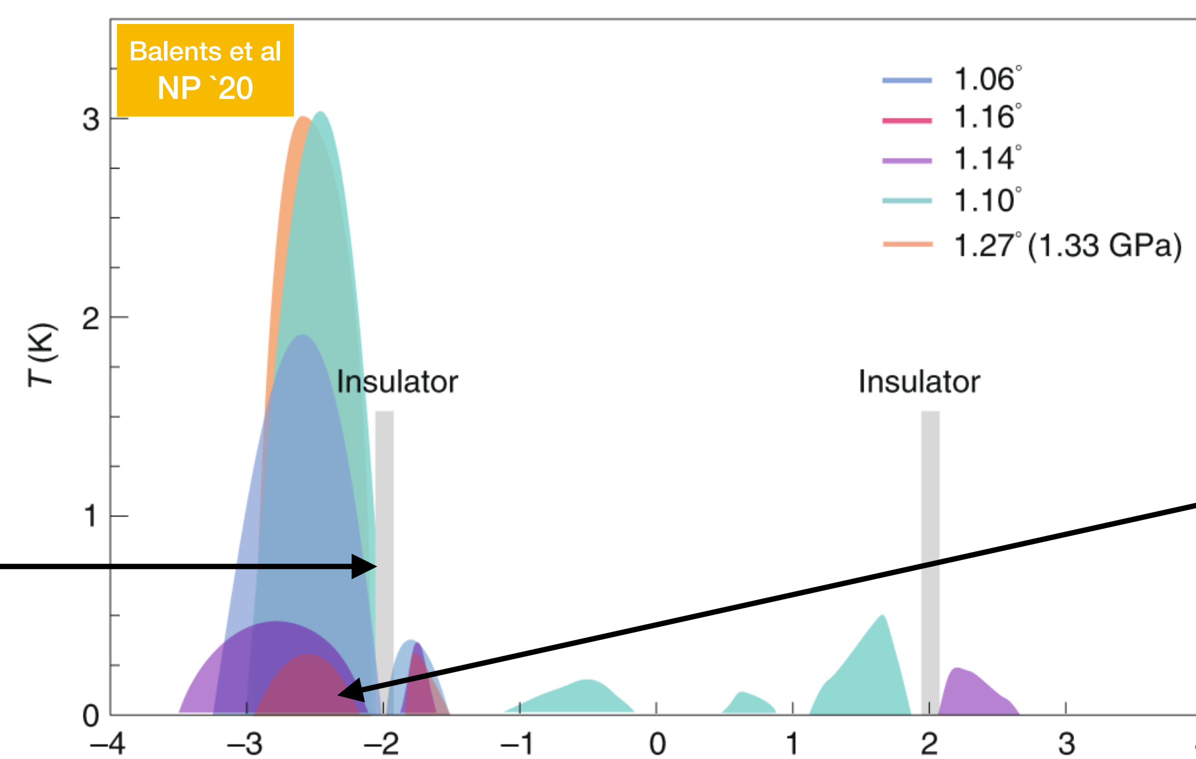
$\theta \sim 1/60$ radians

Electron Count

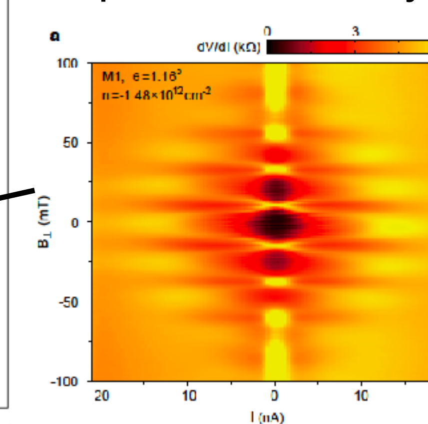


$\nu = +4$

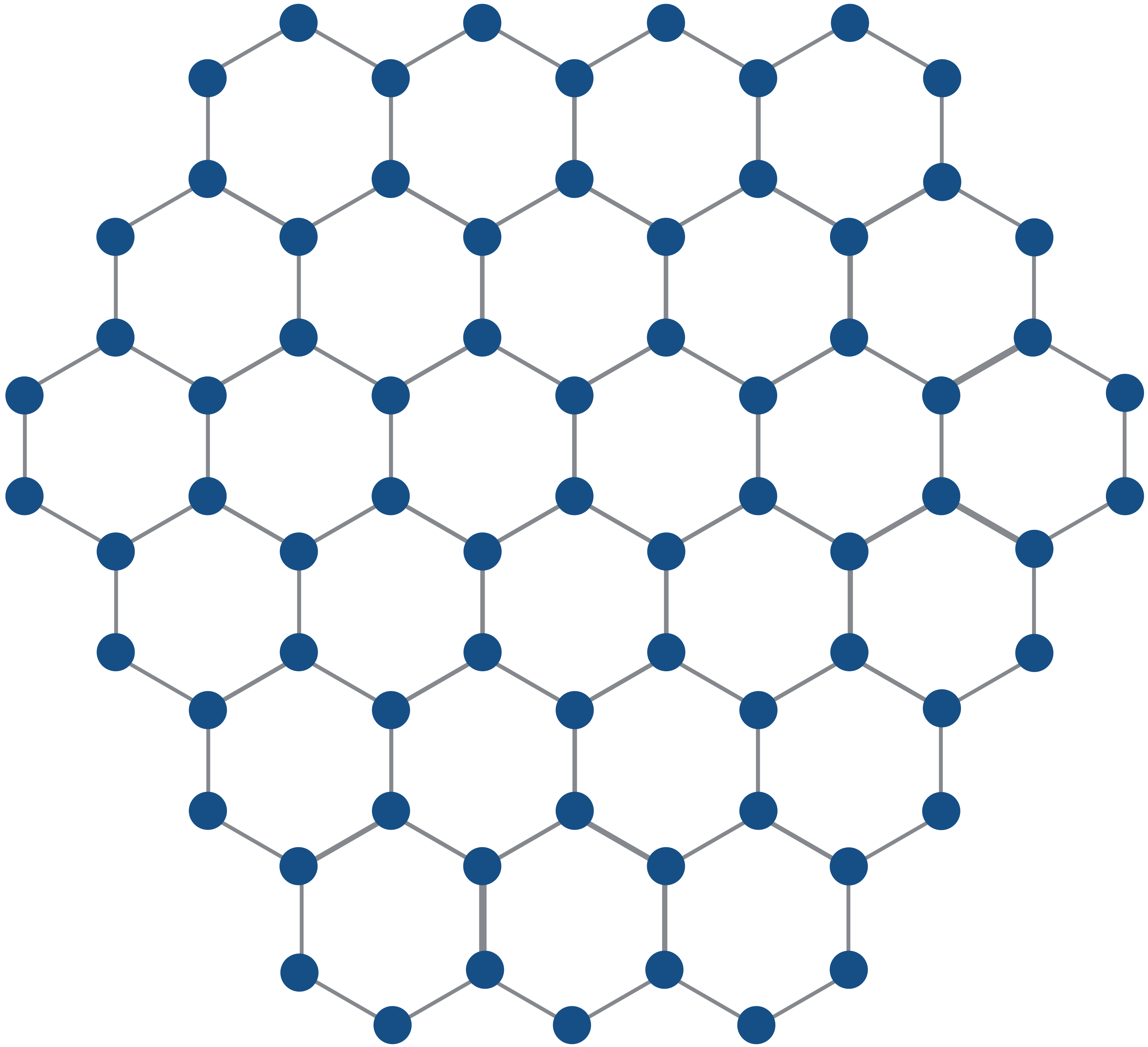
$\nu = -4$



Superconductivity

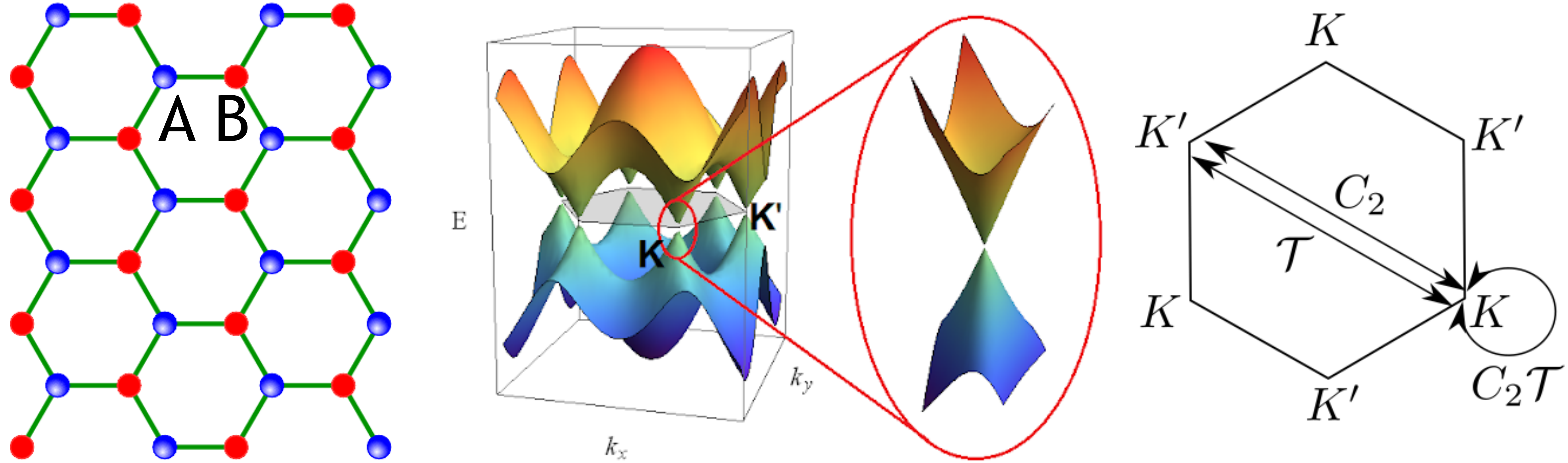


1. Flat Bands



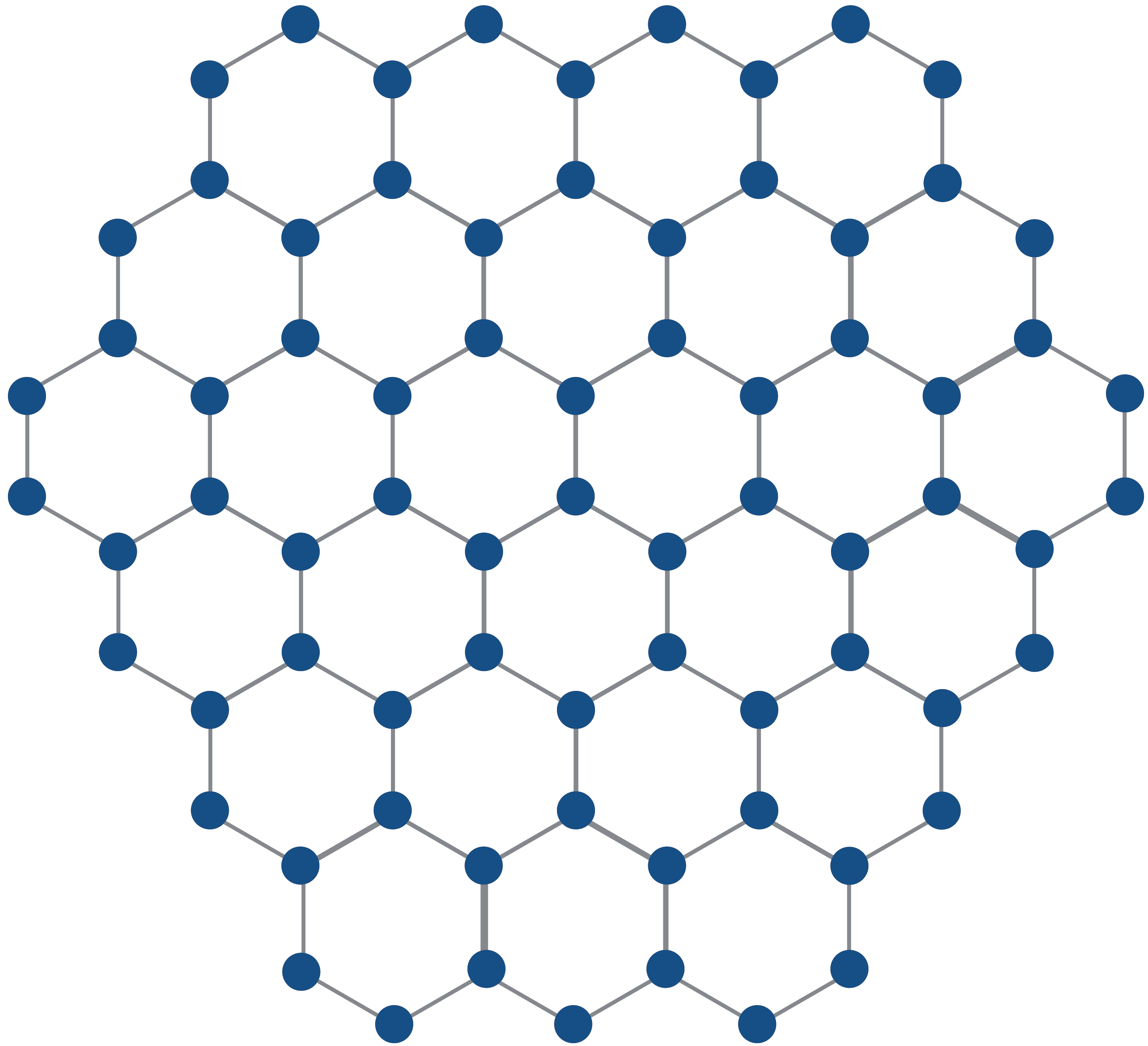
Monolayer

Graphene band structure



Two valleys K, K'

- Time reversal $\mathcal{T}$ and two-fold rotation C_2 exchanges K and K
- $C_2\mathcal{T}$ leaves momentum invariant → no gap at the Dirac points.



Monolayer

Bilayer

Twisted Bilayer

Continuum model

- Larger unit cell→ smaller BZone
- Bistrizer-Macdonald (BM) model (2011)

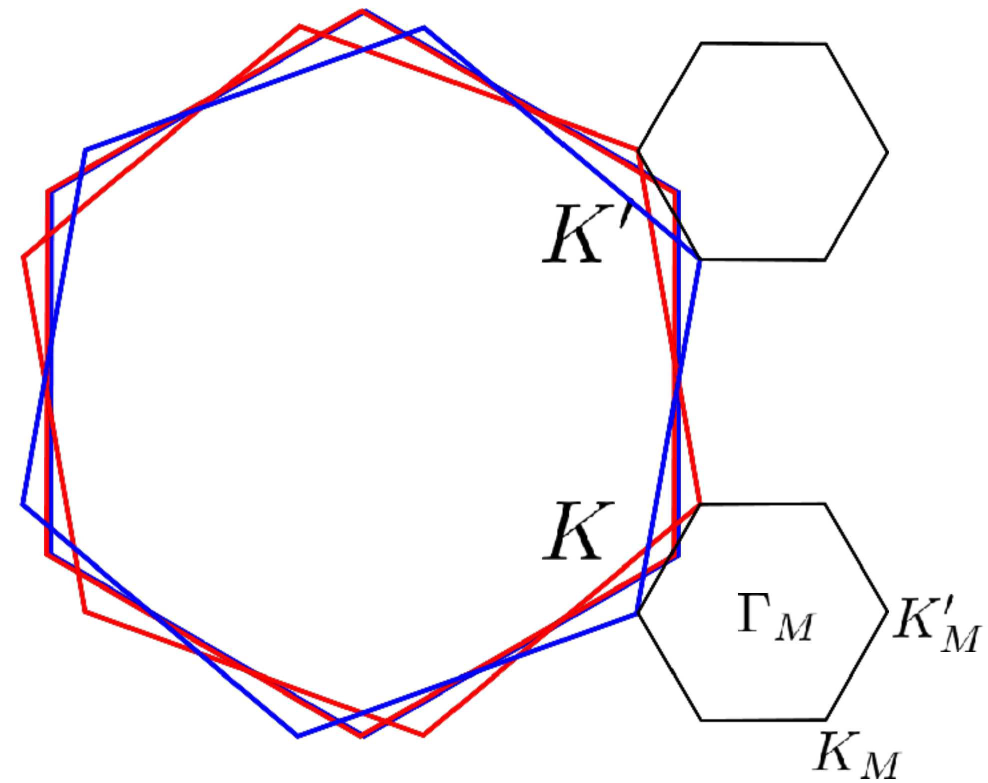
$$\mathcal{H}_K = \begin{pmatrix} -iv_F \boldsymbol{\sigma}_{\theta/2} \cdot \nabla & T(\boldsymbol{r}) \\ T^\dagger(\boldsymbol{r}) & -iv_F \boldsymbol{\sigma}_{-\theta/2} \cdot \nabla \end{pmatrix}_{12},$$

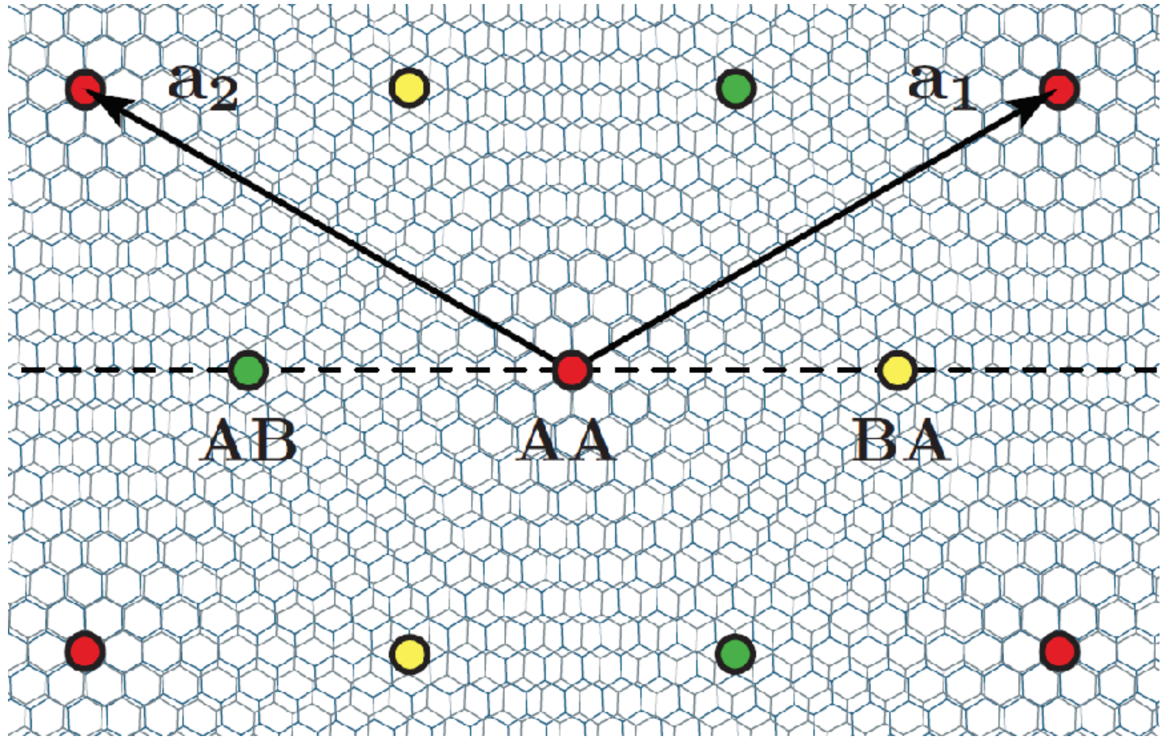
- Moire "potential"

$$T(\boldsymbol{r}) = \begin{pmatrix} w_0 U_0(\boldsymbol{r}) & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & w_0 U_0(\boldsymbol{r}) \end{pmatrix}_{AB}$$

- Lattice relaxation: AB stacking favored to AA stacking (Carr *et al*. 2019, Nam, Koshino 2017)

$$\Longrightarrow \quad w_0/w_1 \approx 0.7$$



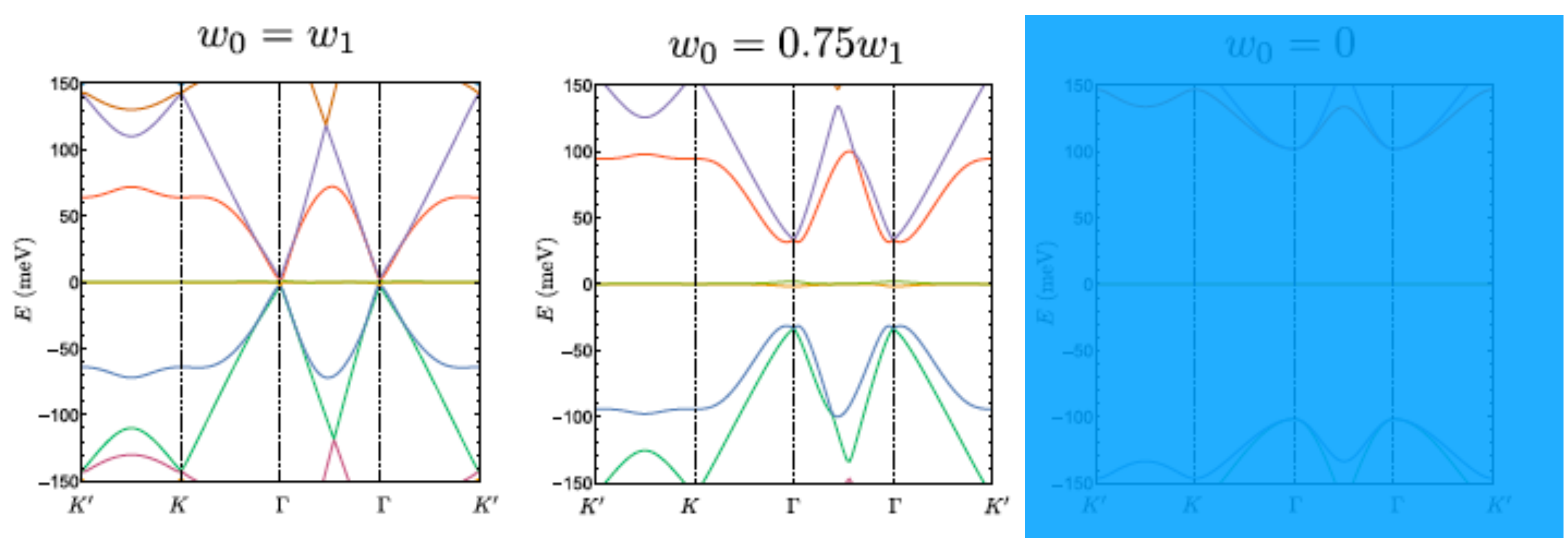


Chiral Model

Tarnopolski, Kruchkov, AV
PRL 2019

Switch off AA coupling. Only AB coupling

$$T(\boldsymbol{r}) = \begin{pmatrix} \cancel{w_0 U_0(\boldsymbol{r})} & w_1 U(\boldsymbol{r}) \\ w_1 U^*(-\boldsymbol{r}) & \cancel{w_0 U_0(\boldsymbol{r})} \end{pmatrix}$$



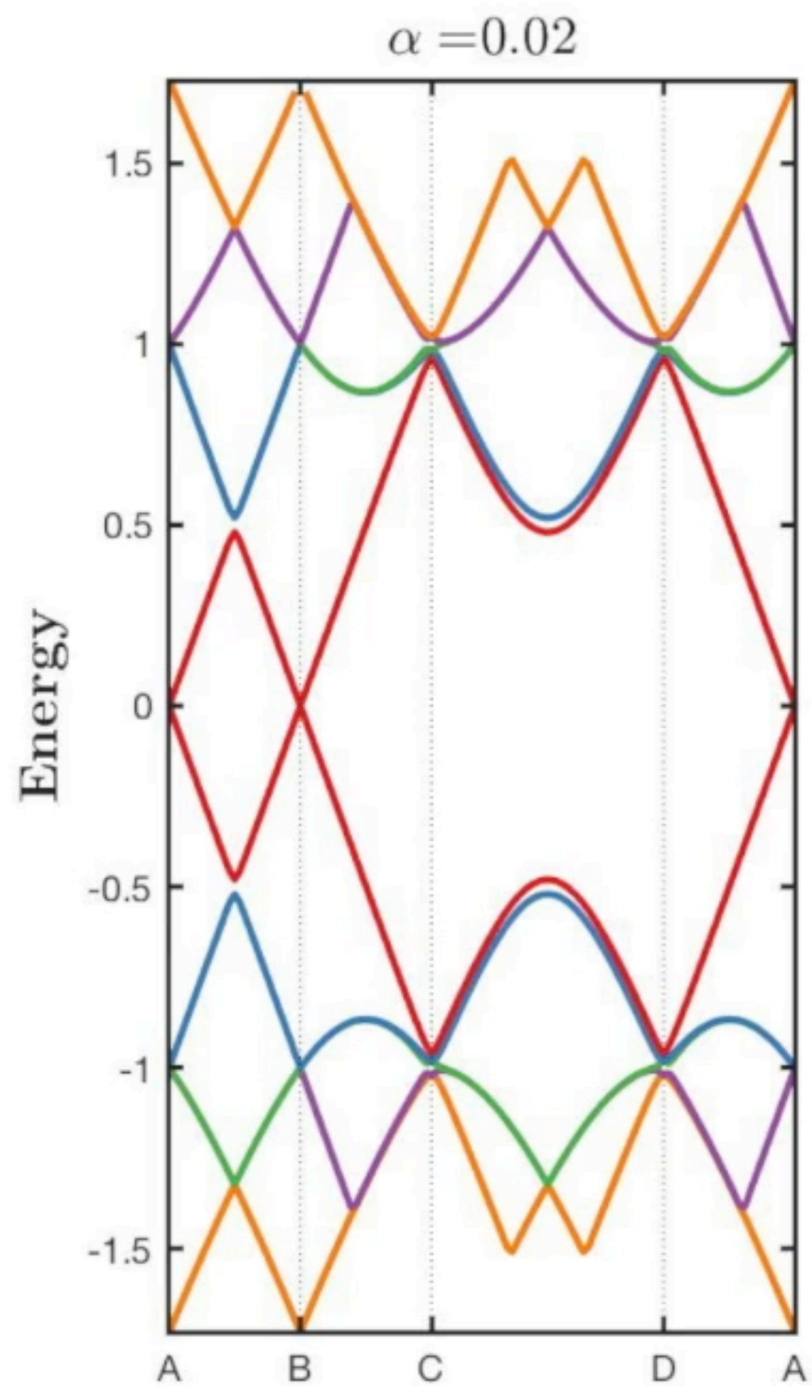
Chiral Symmetry

$\{\sigma_z \otimes 1, \mathcal{H}\} = 0$

sublattice

Perfectly Flat Bands in the Chiral Model

$$\alpha = w_1/(2v_0 k_D \sin(\theta/2))$$



Chiral Model

$$\mathcal{H} = \begin{array}{c} \begin{array}{cc} \mathrm{A} & \mathrm{B} \end{array} \\ \begin{pmatrix} 0 & \mathcal{D}^*(-\mathbf{r}) \\ \mathcal{D}(\mathbf{r}) & 0 \end{pmatrix} \end{array} , \quad \mathcal{D}(\mathbf{r}) = \begin{array}{cc} \begin{array}{cc} \mathrm{U} & \mathrm{D} \end{array} & \\ \begin{pmatrix} -2i\bar{\partial} & \alpha U(\mathbf{r}) \\ \alpha U(-\mathbf{r}) & -2i\bar{\partial} \end{pmatrix} & \begin{array}{c} \mathrm{U} \\ \mathrm{D} \end{array} \end{array}$$

$$\bar{\partial} = \frac{1}{2}(\partial_x + i\partial_y) \qquad U(\boldsymbol{r}) = e^{-i\boldsymbol{q}_1\boldsymbol{r}} + e^{i2\pi/3}e^{-i\boldsymbol{q}_2\boldsymbol{r}} + e^{-i2\pi/3}e^{-i\boldsymbol{q}_3\boldsymbol{r}}$$

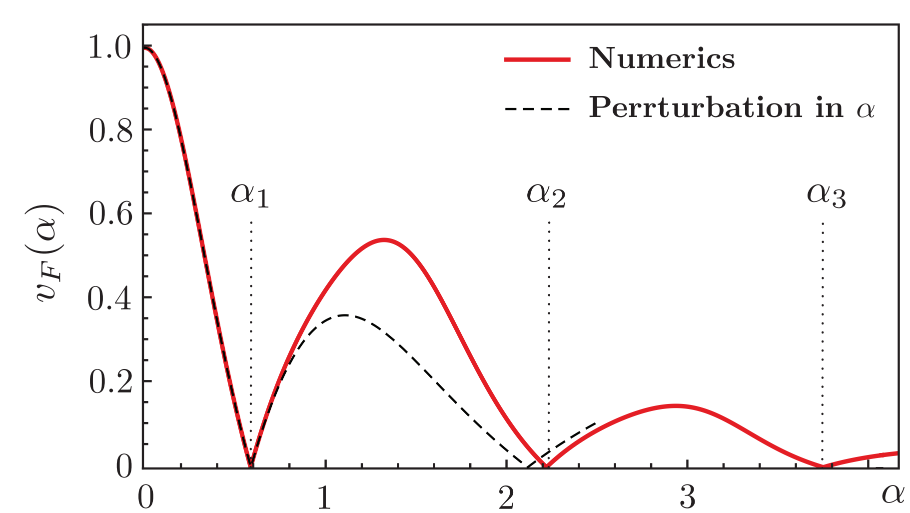
$$\begin{pmatrix} 0 & \mathcal{D}^*(-r) \\ \mathcal{D}(r) & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \psi_B(r) \end{pmatrix} = 0$$

Gives:
perfectly flat bands at a series of magic angles.

Perturbation Theory:

$$\psi_K(\mathbf{r}) = \begin{pmatrix} \psi_{K,1} \\ \psi_{K,2} \end{pmatrix} = \begin{pmatrix} 1 + \alpha^2 u_2 + \alpha^4 u_4 + \dots \\ \alpha u_1 + \alpha^3 u_3 + \dots \end{pmatrix}$$

$$v_F(\alpha) = \frac{1 - 3\alpha^2 + \alpha^4 - \frac{111\alpha^6}{49} + \frac{143\alpha^8}{294} + \dots}{1 + 3\alpha^2 + 2\alpha^4 + \frac{6\alpha^6}{7} + \frac{107\alpha^8}{98} + \dots}$$



Tarnopolski, Kruchkov, AV PRL 2019

Chiral Limit Wavefunctions

Many special properties:

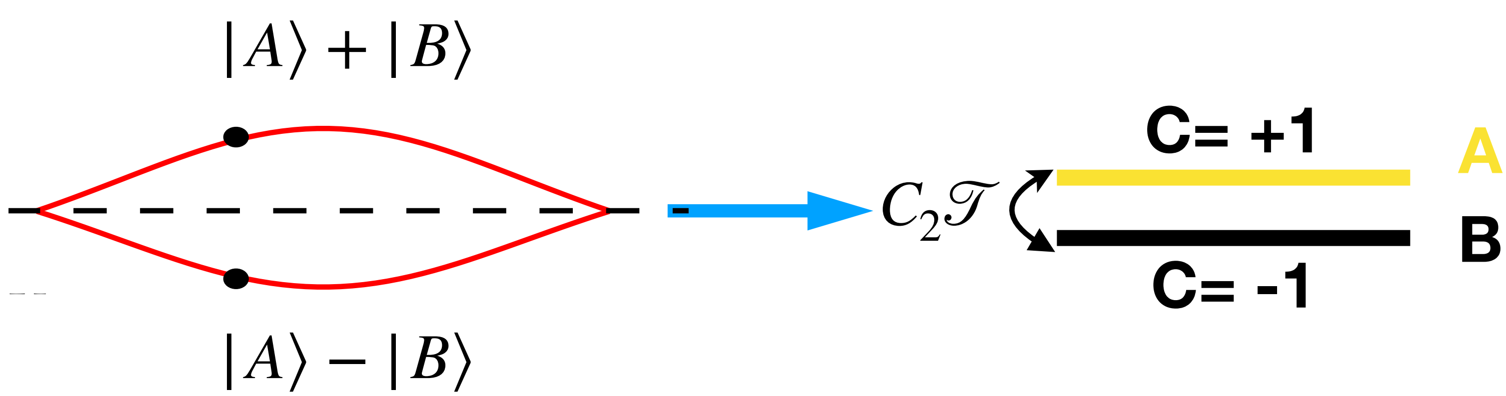
(i) Chern number+ sublattice polarized

(ii) nearly uniform Berry curvature

(iii) Ideal droplet condition - quantum metric is

$$\psi_A(x,y) = \frac{\psi_K(x,y)}{\theta_1(\frac{z-z_0}{a_1}\,|\,\omega)} f(x+iy)$$

$$\eta(k) = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \Omega(k)$$

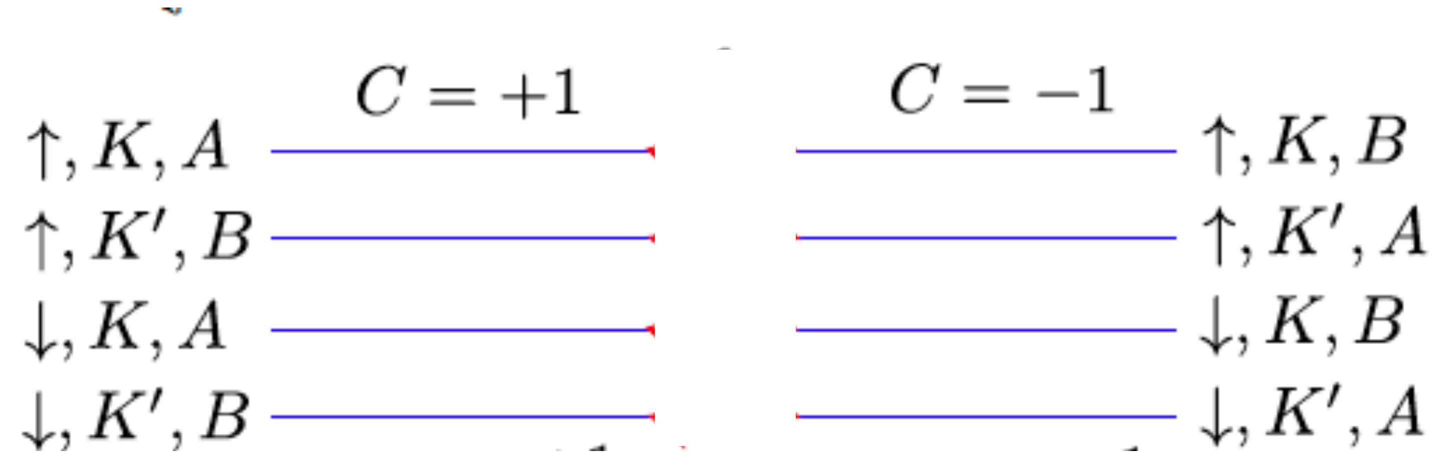


Valley K
Linear combination of bands

Chern =±1;
Sublattice polarized

Ledwith, Tarnopolsky, Khalaf, AV `20
Wang, Zheng, Millis, Cano `20
Becker, Embree, Wittsten, Zworski `20

Interactions



$$\mathcal{H}=\sum_{\boldsymbol{k}} c_{\boldsymbol{k}}^{\dagger} h_{\boldsymbol{k}} c_{\boldsymbol{k}}+\frac{1}{2A}\sum_{\boldsymbol{q}} V_{\boldsymbol{q}} \delta\rho_{\boldsymbol{q}} \delta\rho_{-\boldsymbol{q}}, \quad \delta\rho_{\boldsymbol{q}}=\rho_{\boldsymbol{q}}-\bar{\rho}_{\boldsymbol{q}},$$

$$\rho(q)=\sum_{k\in BZ} c_k^{\dagger} \Lambda_q(k) c_{k+q}$$

Screened Coulomb

Form factor - wave functions of flat bands -plays a key role

PT

Kang, Vafek PRL 2019: SU(4)
Bultnick, Khalaf et al. arXiv:1911.02045

2. Flavor Ordered Insulator

Ground State and Hidden Symmetry of Magic Angle Graphene at Even Integer Filling

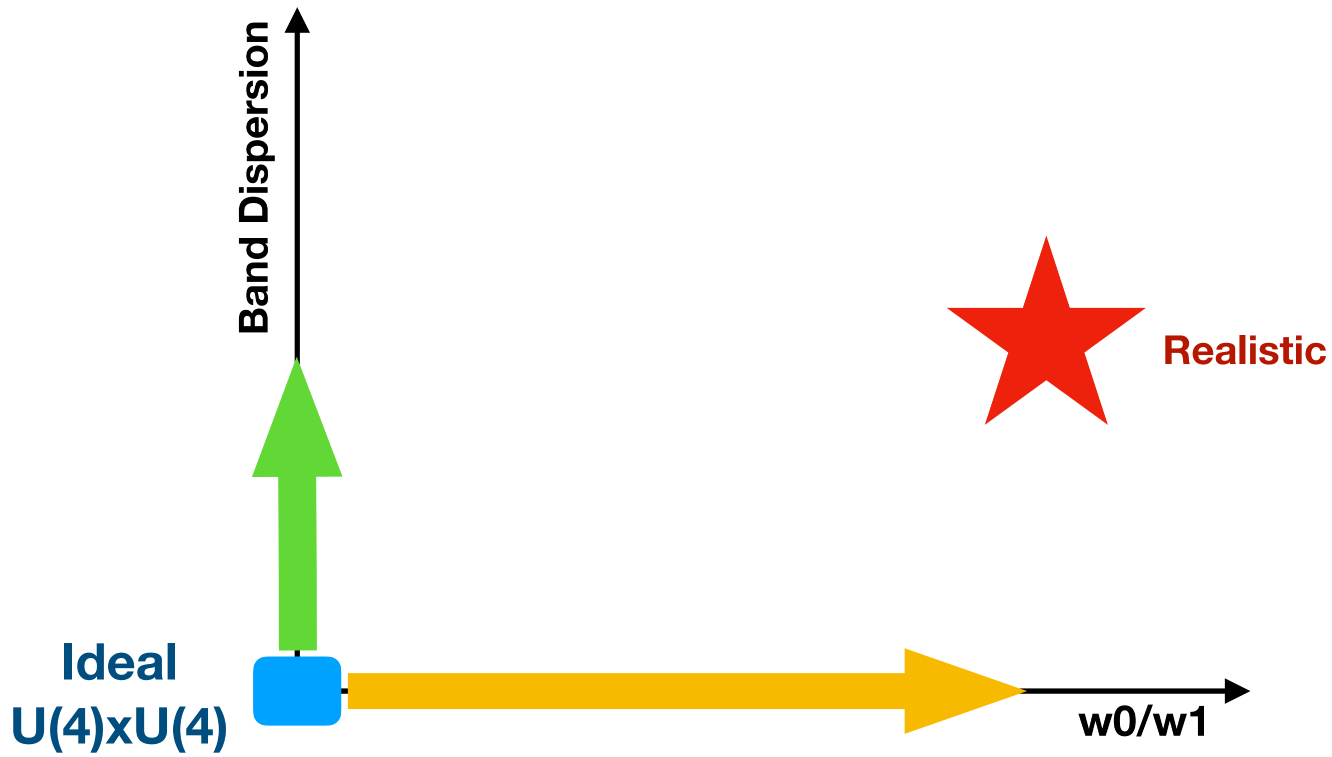
Nick Bultinck,[1,*] Eslam Khalaf,[2,*] Shang Liu,[2] Shubhayu Chatterjee,[1] Ashvin Vishwanath,[2] and Michael P. Zaletel[1,†]

[1]*Department of Physics, University of California, Berkeley, CA 94720, USA*

[2]*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*

arXiv:1911.02045 (PRX 2020)

Correlated Insulators - Ideal Limit

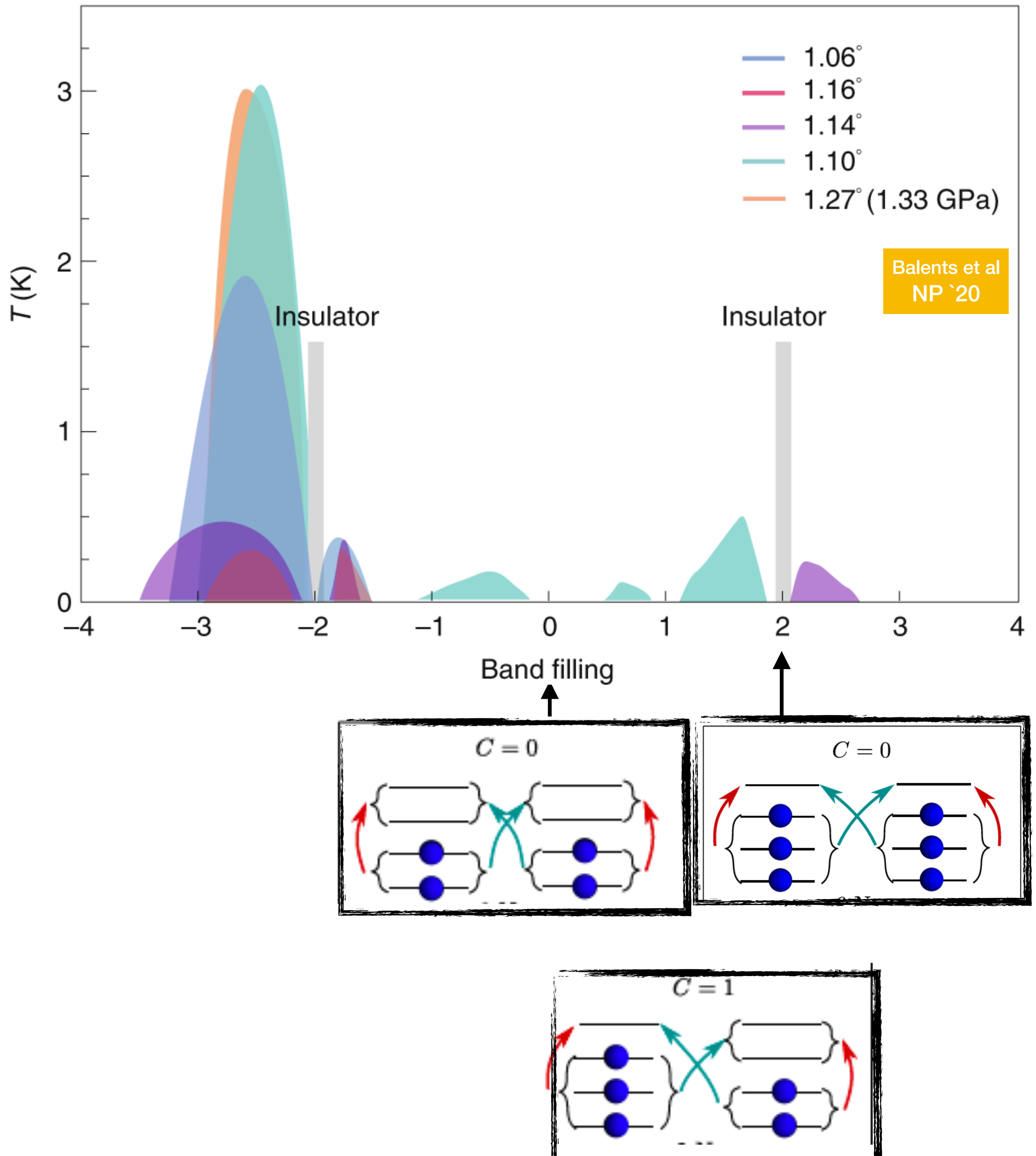


Term	Sublattice-diagonal Int.	Dispersion	Sublattice-off-diagonal Int.
Energy	E_{Coulomb}	$0.2 - 0.3E_{\text{Coulomb}}$	$0.2 - 0.3E_{\text{Coulomb}}$
Symmetry	$\text{U}(4) \times \text{U}(4)$	$\text{U}_R(4)$	$\text{U}(4)_R$

Density:

$$\rho \approx \rho_{C=+1} + \rho_{C=-1}$$

Generalized Flavor Ferromagnets



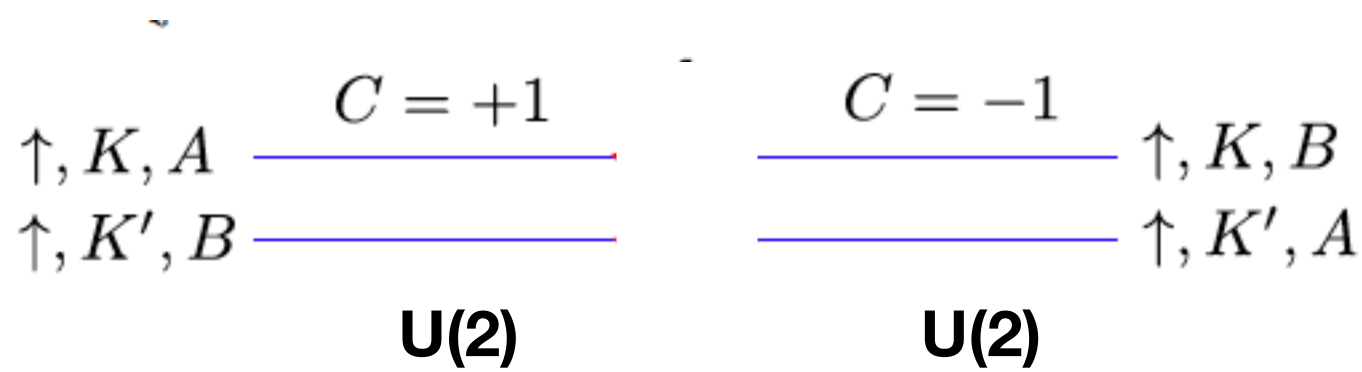
$$Q_+ = 4 - \sum |z^+_{filled}\rangle\langle z^+_{filled}|$$

$$Q_- = 4 - \sum |z^-_{filled}\rangle\langle z^-_{filled}|$$

Eslam Khalaf, Bultinck, *AV,* Zaletel arxiv:2009.14827
Kang and Vafek arXiv:2009.09413
Kumar, Xie, MacDonald arXiv:2010.05946
Bernevig, Lian, Cowsik; Xie, Regnault, Song arXiv:2009.14200

Simplified Model - Spinless TBG

$$\mathcal{H}_{\text{int}} = \frac{1}{2A}\sum_{q} V_{q}\delta\rho_{q}\delta\rho_{-q},$$



No Dispersion & Chiral limit:

Density: $\rho \approx \rho_{C=+1} + \rho_{C=-1}$

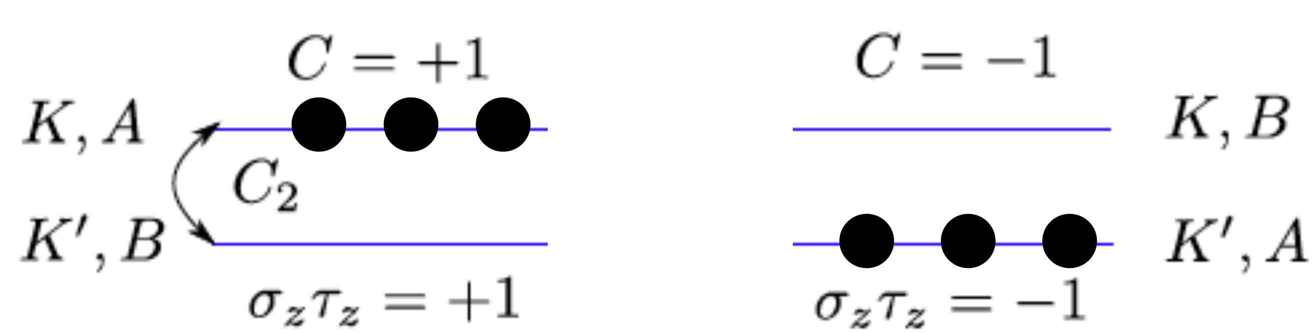
(Spinless Model) **Or** $\nu = \pm 2$

- Family of exact ground states - generalized ferromagnets. Fill Chern Bands.

Argument:

$V_q \geq 0$ and

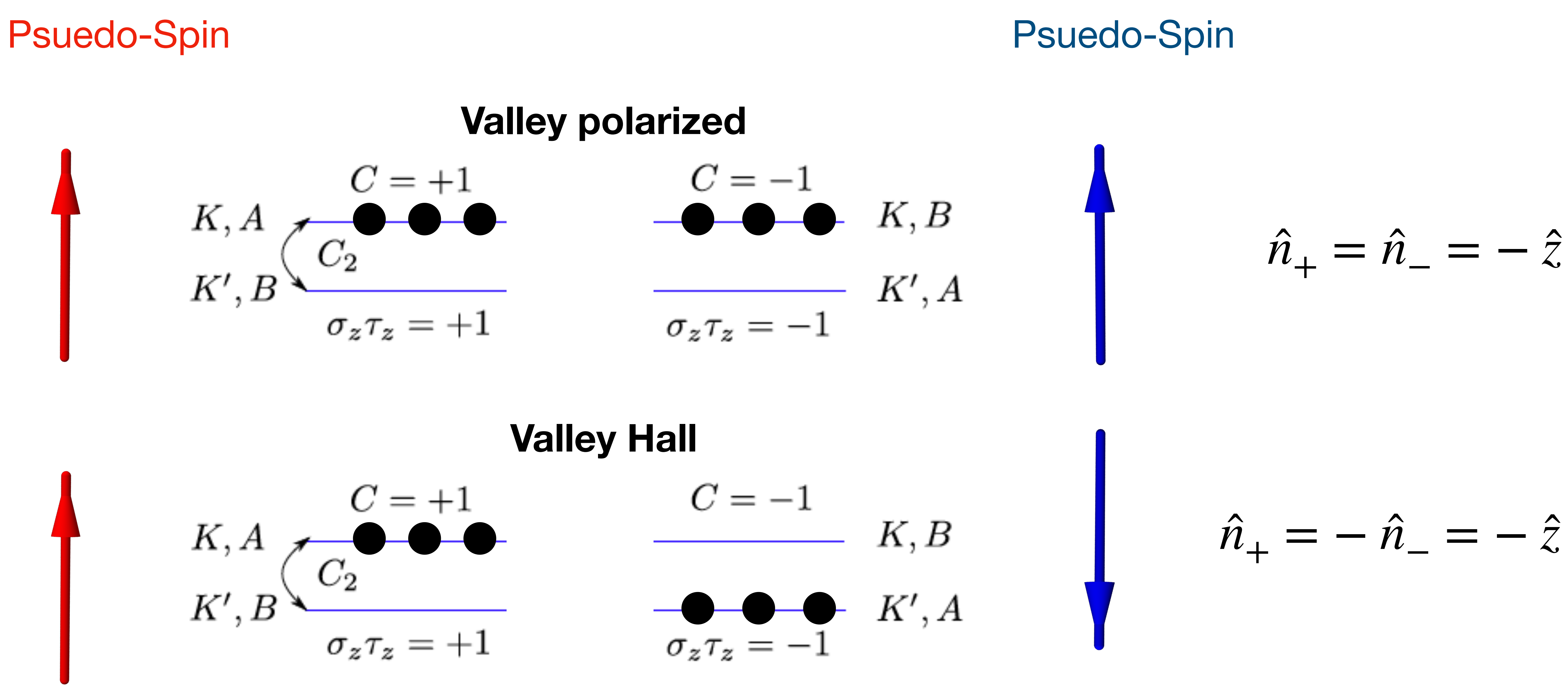
$\delta\rho_q|\Psi\rangle = 0$



Ground States of Ideal Model

$$Q_+ = 1 - |z_+\rangle\langle z_+| = \sigma \cdot \hat{n}_+$$

$$Q_- = 1 - |z_-\rangle\langle z_-| = \sigma \cdot \hat{n}_-$$

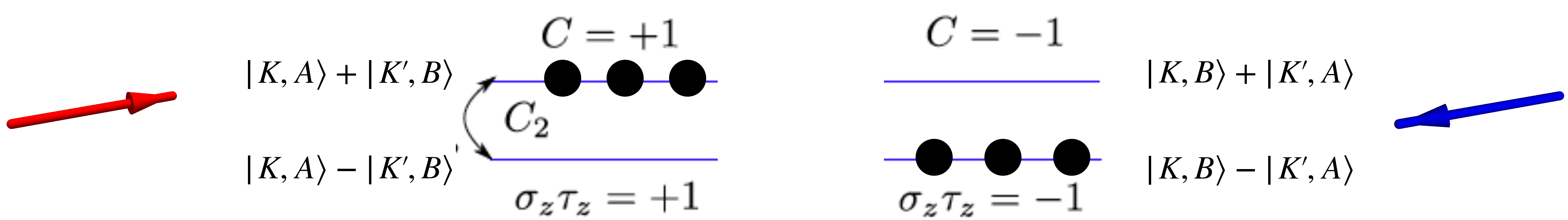


Ground States of Ideal Model

- Intervalley coherent (IVC) states break valley U(1): translation symmetry at graphene scale.

Psuedo-Spin

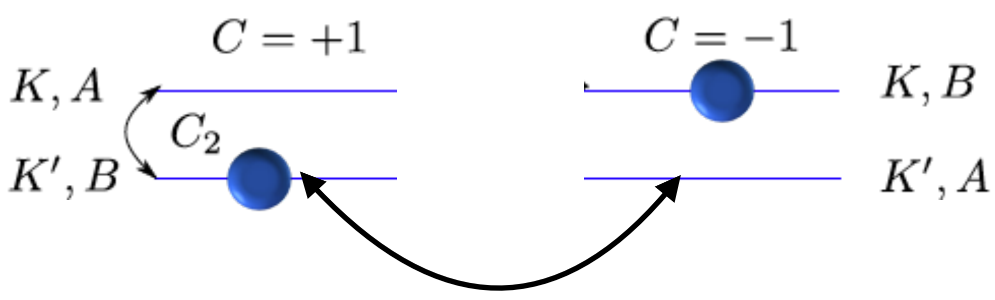
Psuedo-Spin

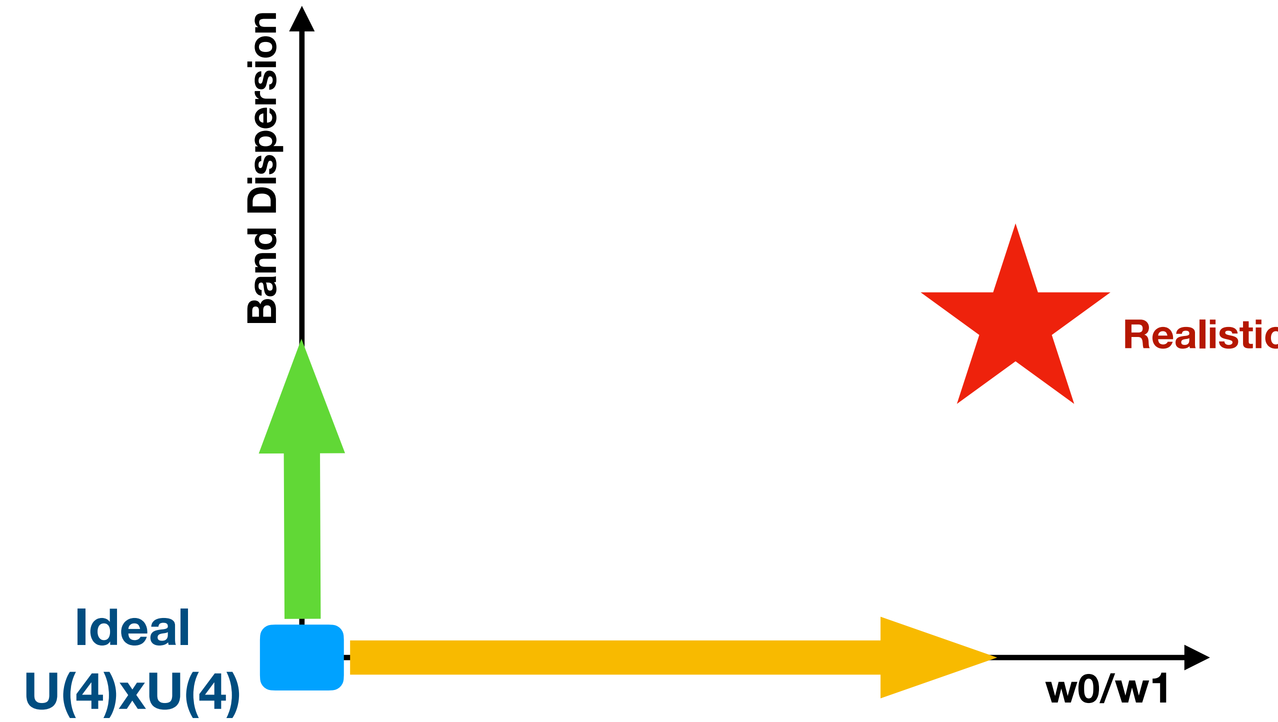


$$\hat{n}_+ = \hat{n}_- = -\hat{x}$$

K-IVC

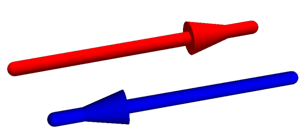
Effect of Dispersion & W_0



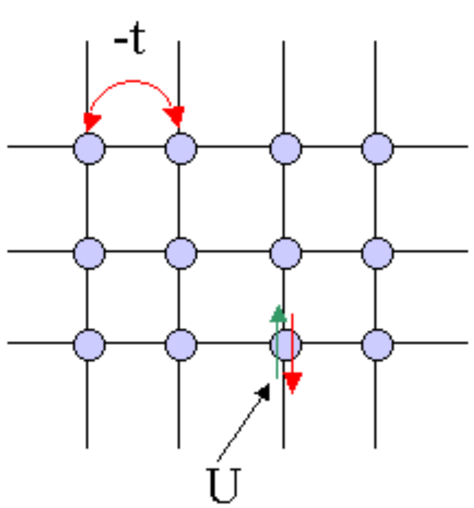


$$+\lambda(\mathbf{n}_+ \cdot \mathbf{n}_- - 2n^z_+ n^z_-)$$

anisotropy - moving from chiral limit



K-IVC

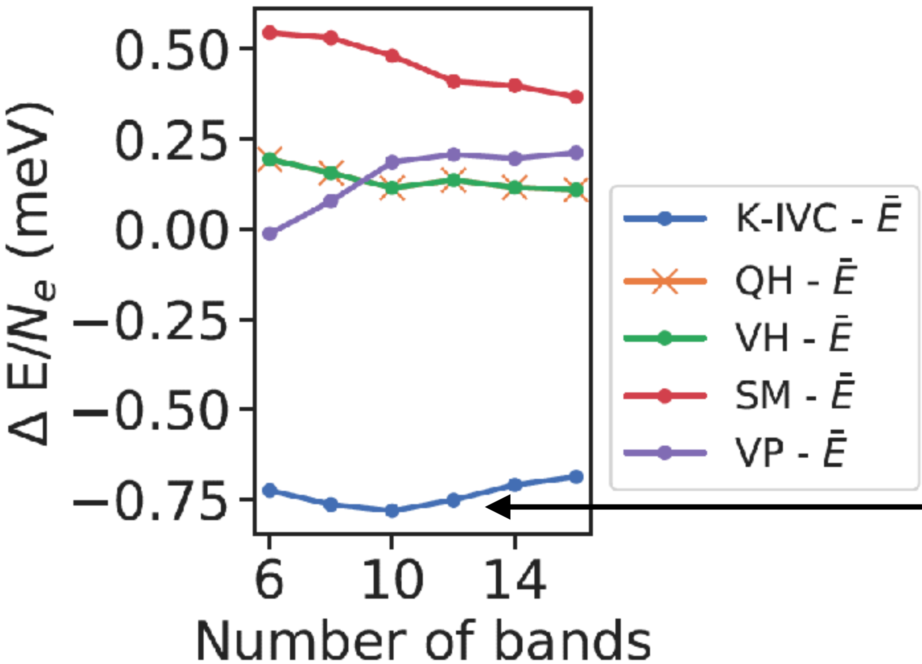


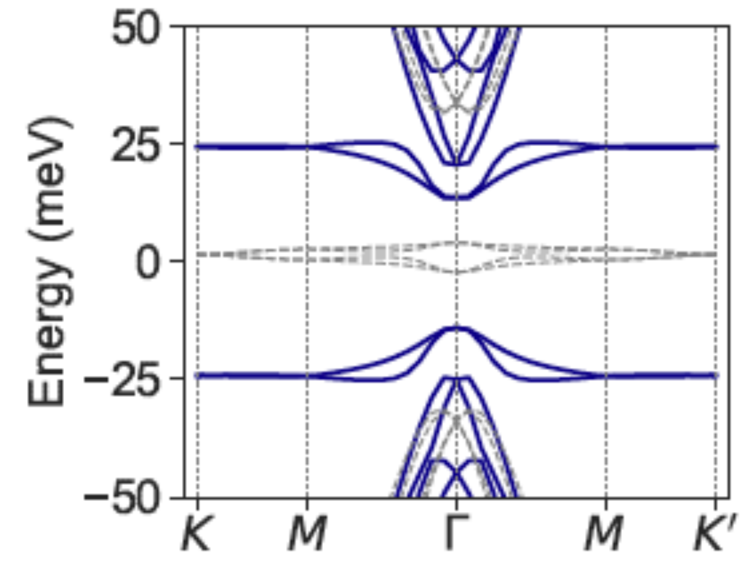
J~ $4t^2/U$

Superexchange

Bultnick et al. arXiv:1911.02045
Kang and Vafek `19

Hartree Fock & DMRG Numerics -Confirms This Picture

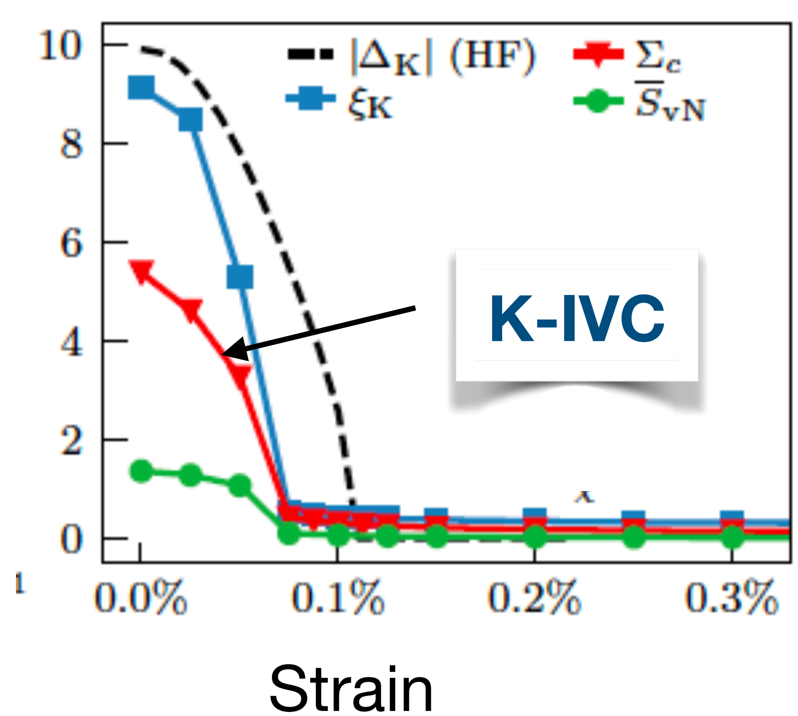


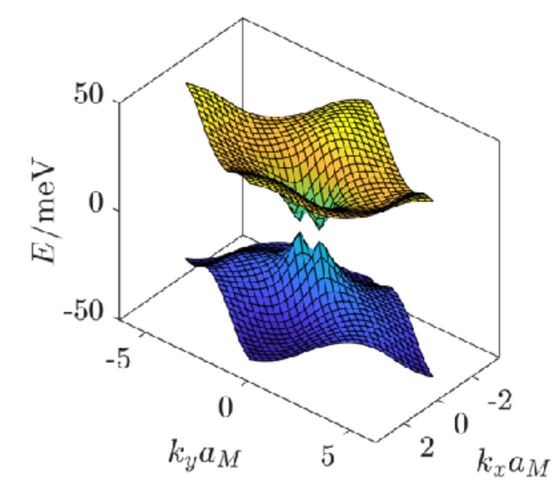


$\theta = 1.05^{\mathrm{o}}\, w_0 = 80\mathrm{meV};\, w_1 = 110\mathrm{meV};\, \epsilon = 7$

$$Q = \sigma_y \left(\Delta_R \tau_x + \Delta_I \tau_y \right)$$

Bultnick et al. arXiv:1911.02045
Kang and Vafek `19





Competing state - *nematic* semimetal.

Shang Liu et al. arXiv:1905.07409
Kang& Vafek `19
Parker et al. arXiv:2012.09885

Kramers IVC - Properties

U(1)$_{valley}$ spontaneously broken -
Goldstone modes (lattice translation)

K-IVC

$$Q = \sigma_y \left(\Delta_R \tau_x + \Delta_I \tau_y \right)$$

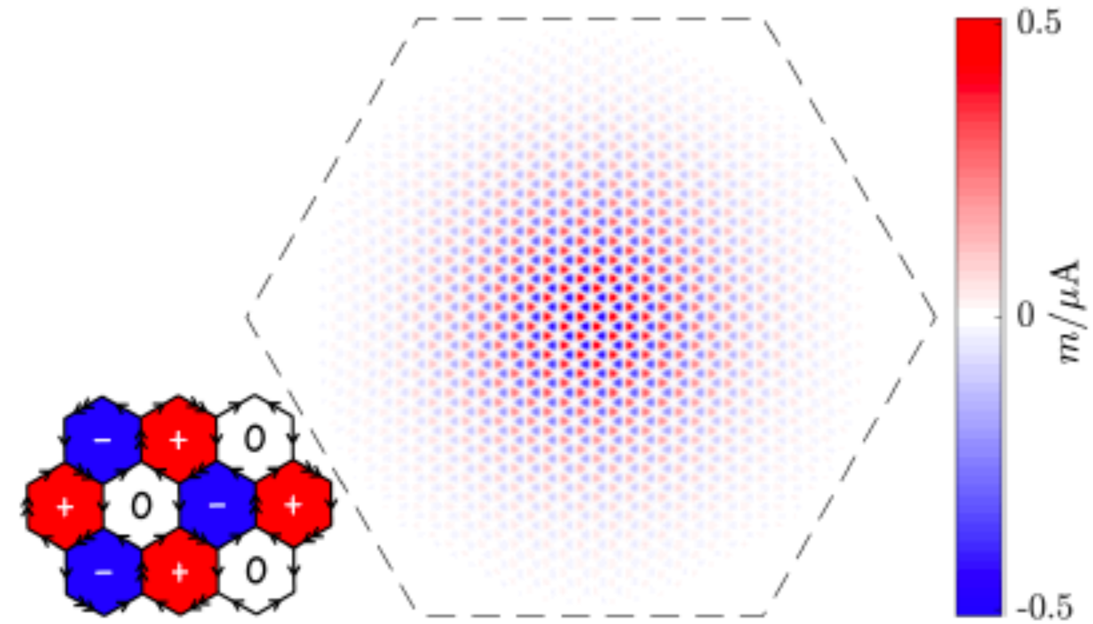
- Spontaneously breaks $\mathcal{T}$, but preserves a combination of $\mathcal{T}$ and π valley rotation $\mathcal{T}' = \tau_z \mathcal{T} = \tau_y \mathcal{K}$.

$$\mathcal{T}'^2 = -1$$

Same symmetries as topological insulator

Involves opposite Chern number band

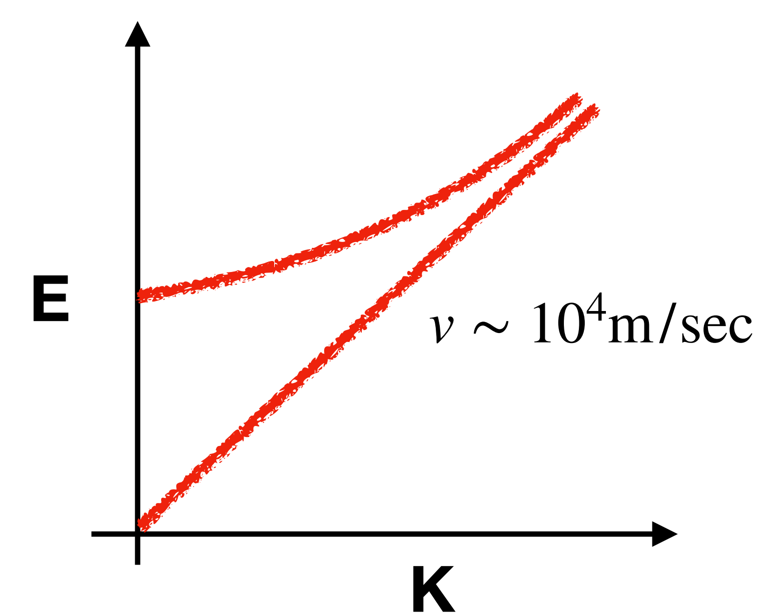
Nontrivial Z_2 topology!



Current pattern*

“Spontaneous” topological insulator

(Topological “Mott” Insulator - Raghu, Qi, Honerkamp, Zhang)



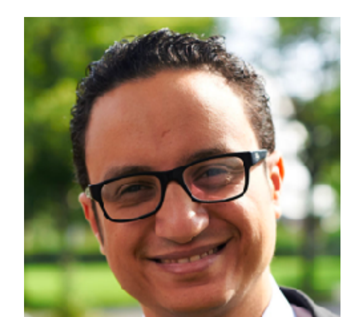
Edge states? Requires `smooth’ edge

*Chakravarty, Laughlin, Morr, Nayak. Zhu, Aji, Varma.

Acknowledgements

- Flat Band Topology and Geometry
    - Hoi Chun (Adrian) Po, Liujun Zou, T. Senthil,
    - Grisha Tarnopolski, Alex Kruchkov, Patrick Ledwith
- Correlated Insulators and Nematic Semimetal
    - (Harvard) Eslam Khalaf, Shang Liu, Jong Yeon Lee
    - (Berkeley) Nick Bultnick, Shubhayu Chatterjee, Mike Zaletel

Patrick Ledwith



Eslam Khalaf

Lecture Notes for More Technical Details:

https://scholar.harvard.edu/avishwanath/teaching