Non-Local Order Parameters

Ashvin Vishwanath
Harvard University

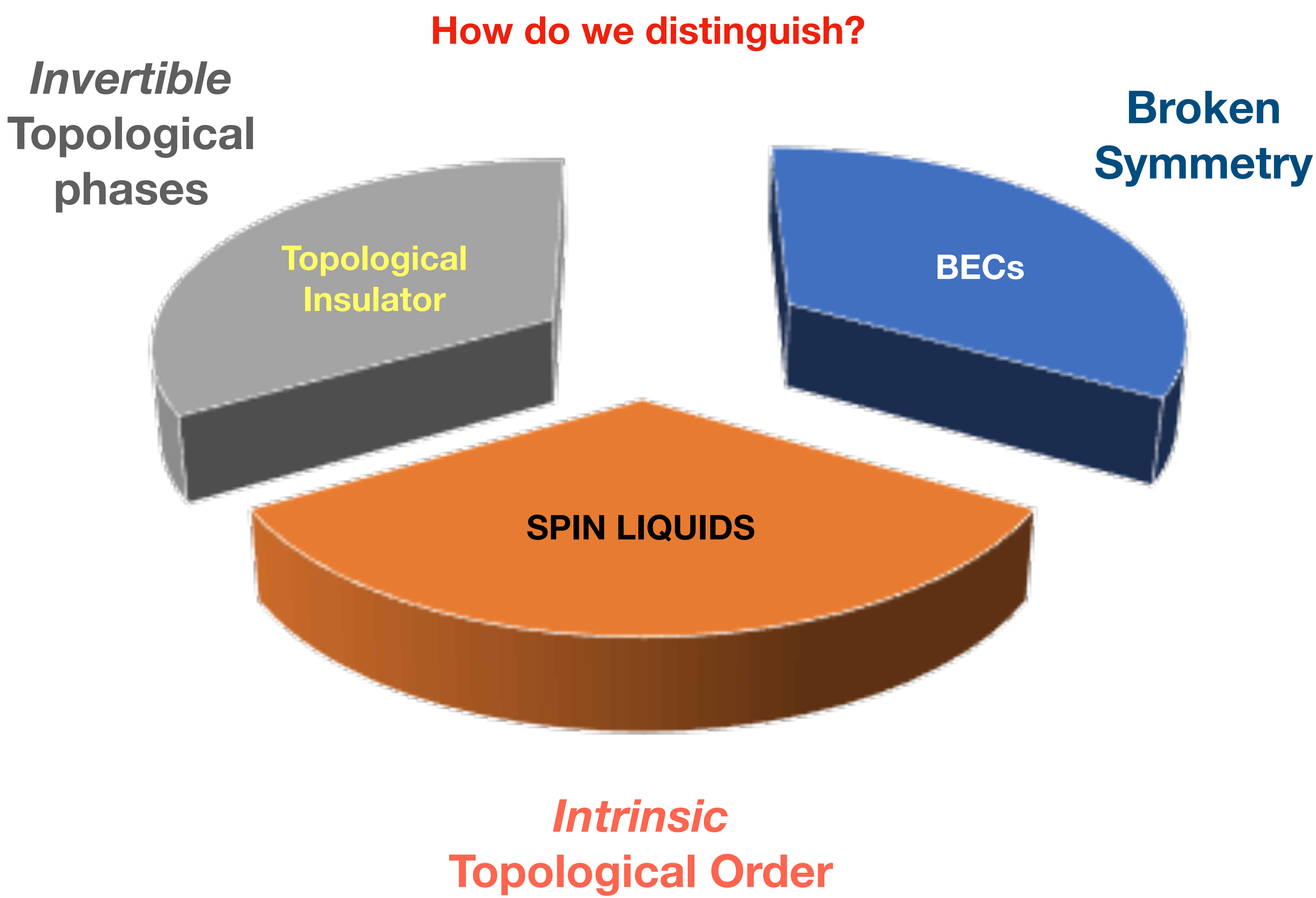
Acknowledgements:

Ruben Verresen
Harvard

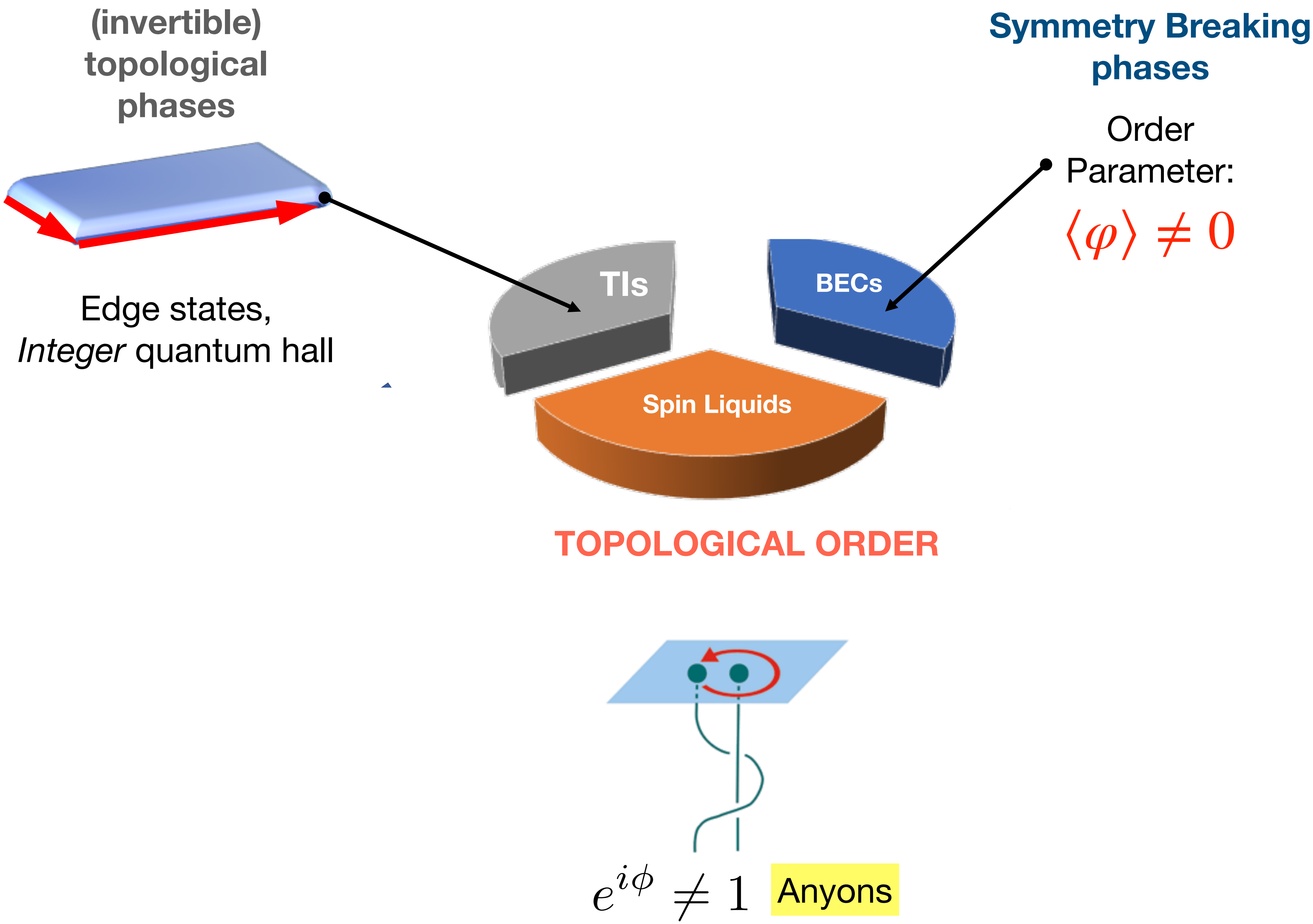
OUTLINE

- Part 0: Quantum states of Matter and observables in *many body* quantum systems.
- Part 1: Condensates, Order parameters and disorder operators.
- Part 2: Symmetry Protected Topological phases and their string orders.
- Part 3: *intrinsic* topological order, spin liquids and their detection.

Quantum Phases of Matter



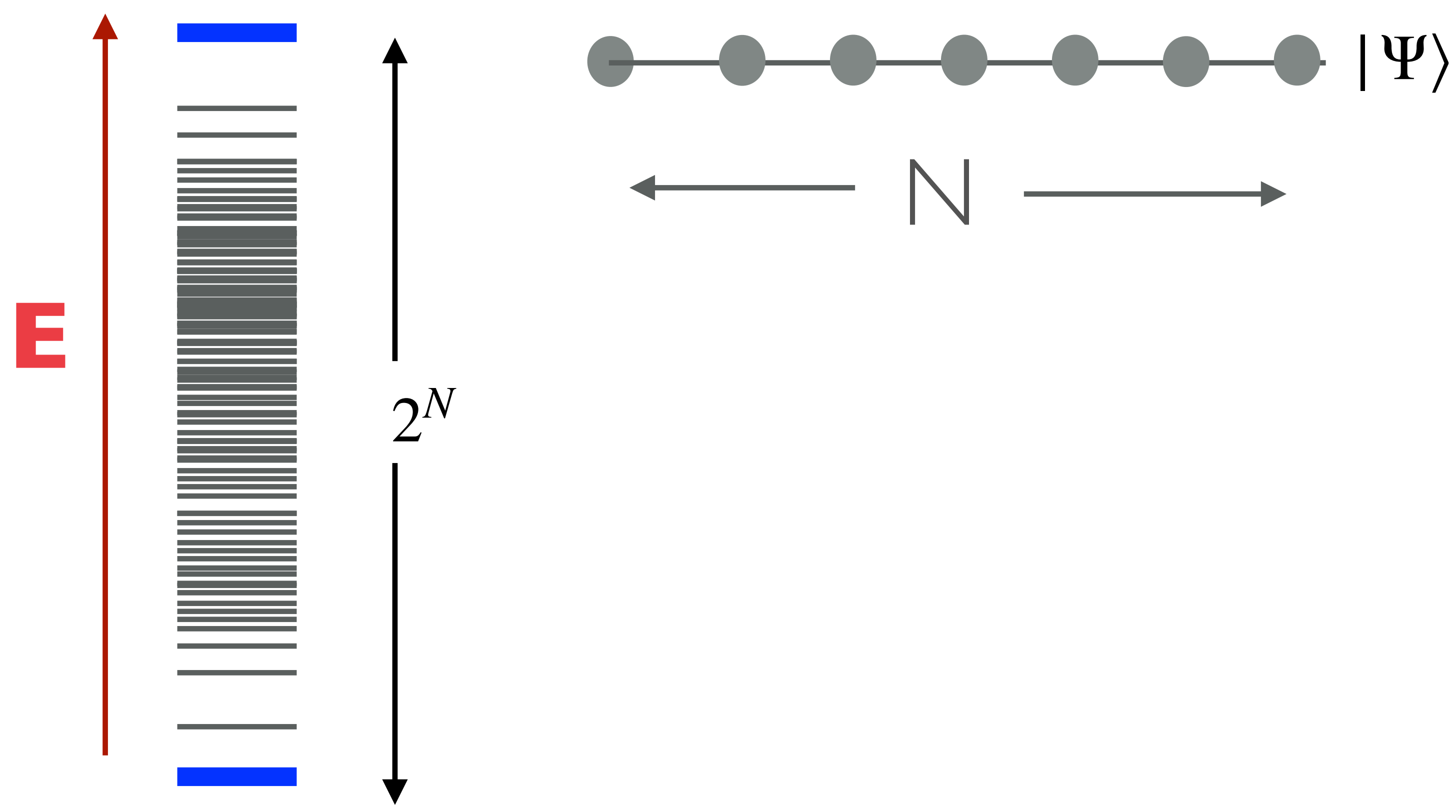
Distinguishing Quantum Phases of Matter



What is the Many Body Hilbert Space?

Hilbert space: John von Neumann coined the term for the abstract concept

Reportedly, **Hilbert** asked **John von Neumann** "but what is a Hilbert space, really?"



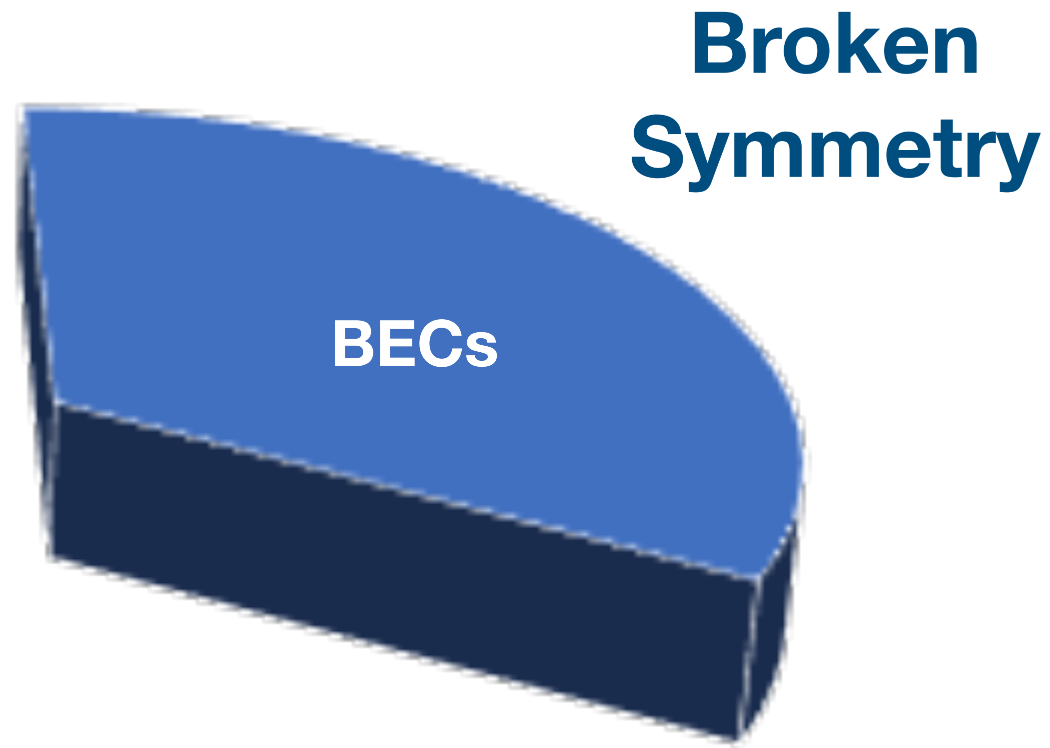
$|\uparrow\rangle \otimes |\uparrow\rangle$ $\quad |\downarrow\rangle \otimes |\uparrow\rangle$

$|\uparrow\rangle \otimes |\downarrow\rangle$ $\quad |\downarrow\rangle \otimes |\downarrow\rangle$

- Here, nonlocal correlators

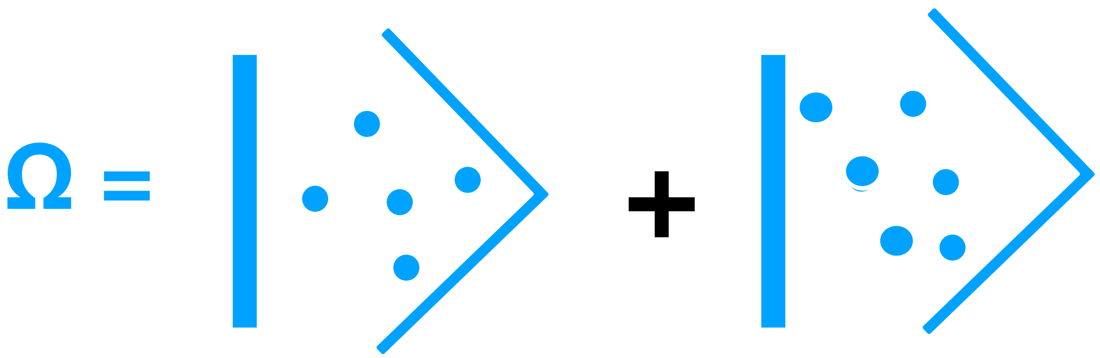
$$S_r = \langle \Psi | Z_r\, Z_{r-1}, Z_{r-2} \ldots Z_2 Z_1 Z_0 | \Psi \rangle$$

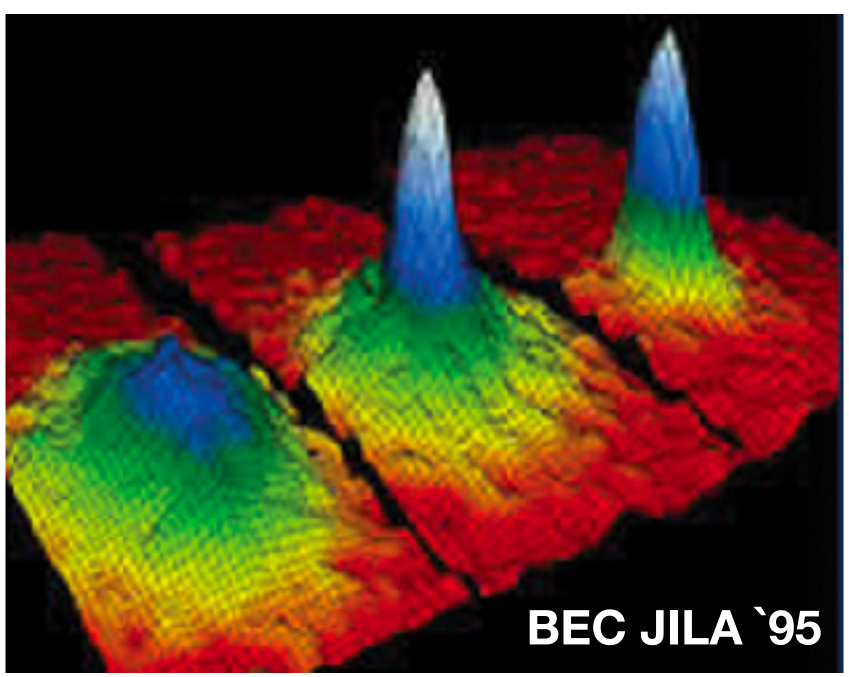
Part 1: Ordered Phases



Measuring Ordered Phases

Bose Einstein Condensate





Condensate

$$\langle b_k^\dagger b_k \rangle = n_{k=0} = f N_{boson}$$

$$\Longrightarrow \lim_{r \to \infty} \langle b_r^\dagger \, b_0 \rangle = \varphi$$

Long Range Order

Since:

$$b_k = \frac{1}{\sqrt{N}} \sum_r e^{i\mathbf{k}\,\mathbf{r}} b_r$$

and:

$$\langle b_{k=0}^\dagger b_{k=0} \rangle = \frac{1}{N} \sum_{r,r'} \langle b_r^\dagger b_{r'} \rangle$$

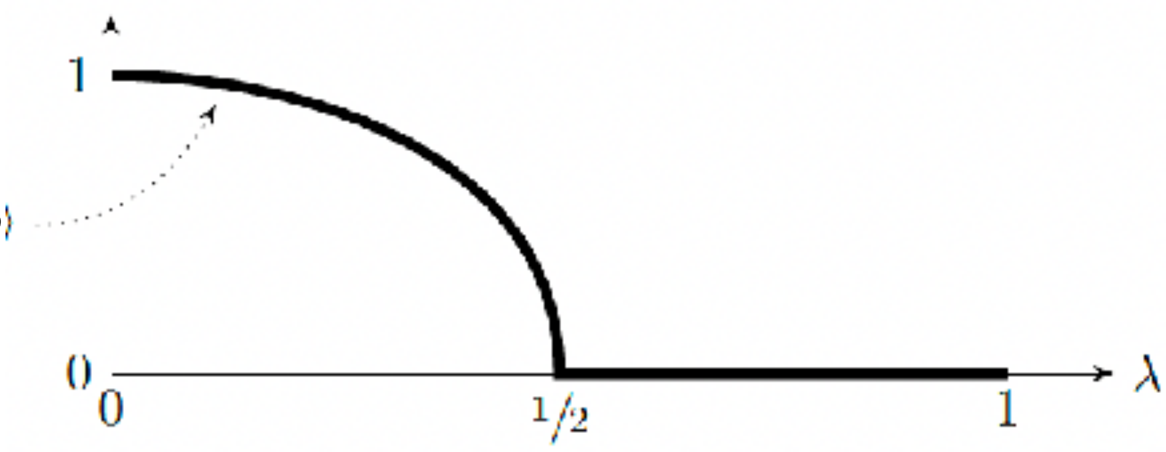
Transverse Field Ising Model

$$H_{ising} = -(1-\lambda)\sum_i Z_i Z_{i+1} - \lambda \sum_i X_i$$

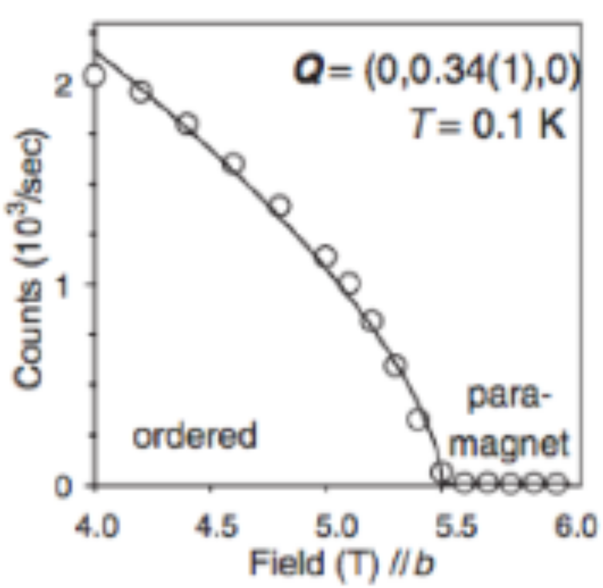
Z_2 Symmetry $= \prod_i X_i$

$$Z_i \to -Z_i$$

$$\lim_{|n-m|\to\infty} \langle Z_n Z_m \rangle \to \varphi$$



Measured by Neutron Scattering



$CoNb_2O_6$

Coldea et al. `11

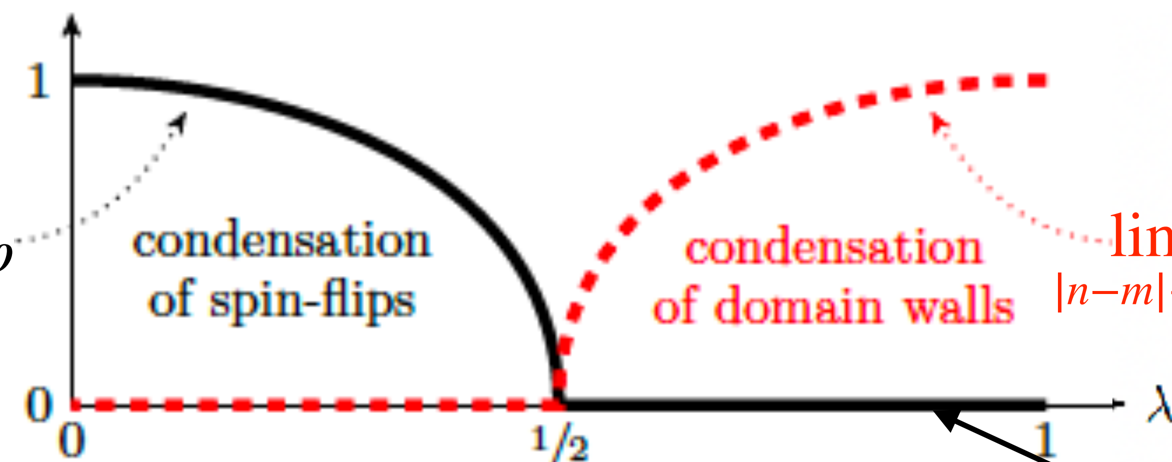
Transverse Field Ising Model

$$H_{ising} = -(1-\lambda)\sum_i Z_i Z_{i+1} - \lambda \sum_i X_i$$

Z2 Symmetry $= \prod_i X_i$

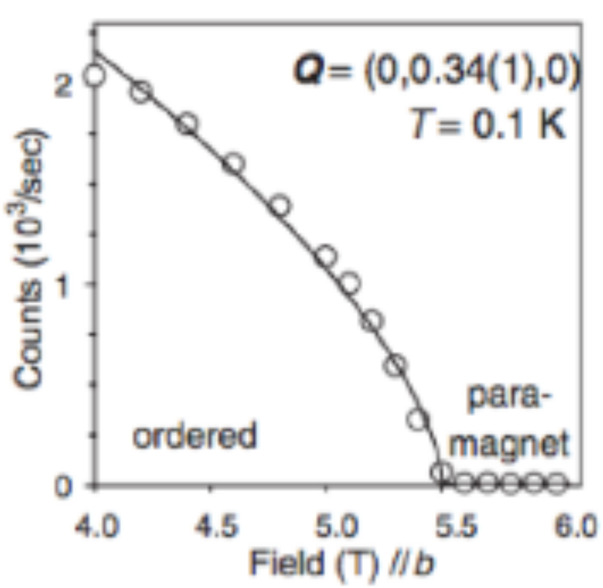
$$Z_i \to -Z_i$$

$$\lim_{|n-m|\to\infty} \langle Z_n Z_m \rangle \to \varphi$$



$$\lim_{|n-m|\to\infty} \langle X_n X_{n-1} \ldots X_{m+1} X_m \rangle \to \tilde{\varphi}$$

Measured by Neutron Scattering



$CoNb_2O_6$

Coldea et al. `11

Can we measure?

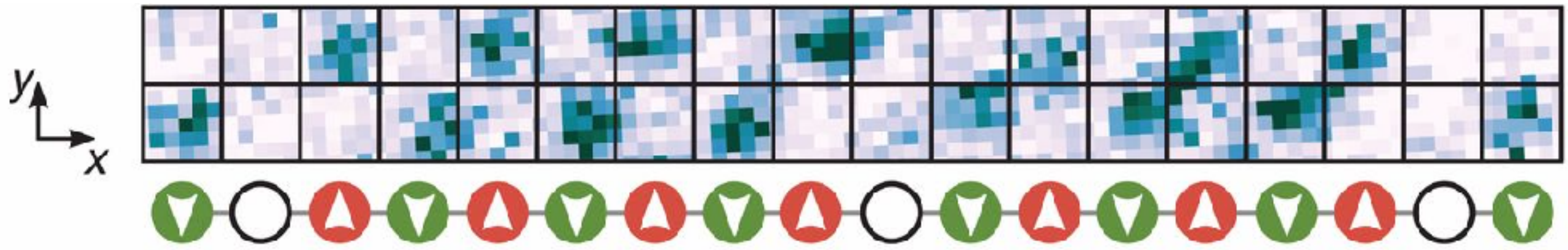
String order

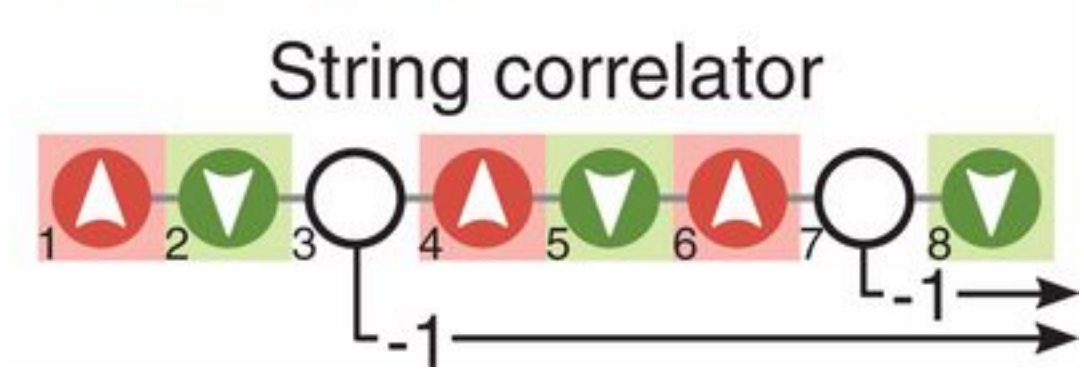
Corresponds to restricting Symmetry to subregion.

Creates domain walls

Measure “domain wall” condensate

Yes! Cold Atom Platforms/NISQC



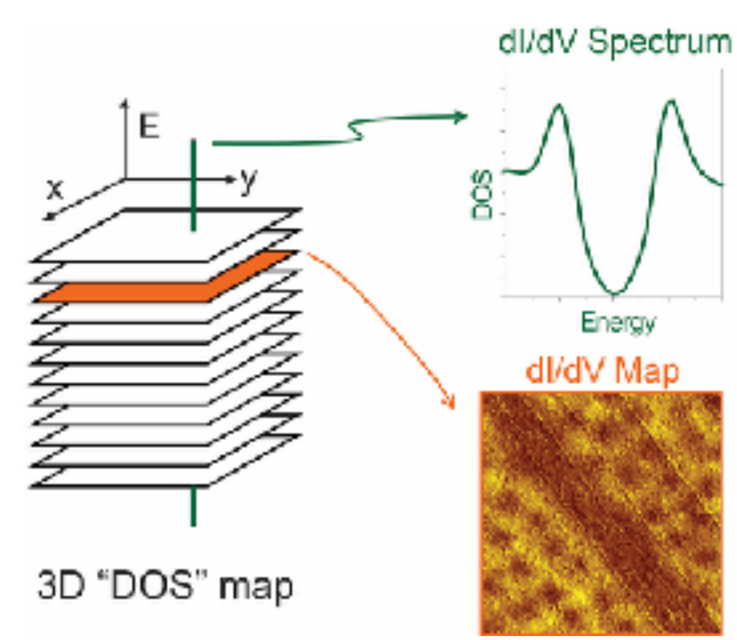


Quantum Gas Microscope

Hilker et al. Science '17

String correlator involves many operators

$$\lim_{|n-m|\to\infty} \langle X_n X_{n-1} \ldots X_{m+1} X_m \rangle \to \tilde{\varphi}$$



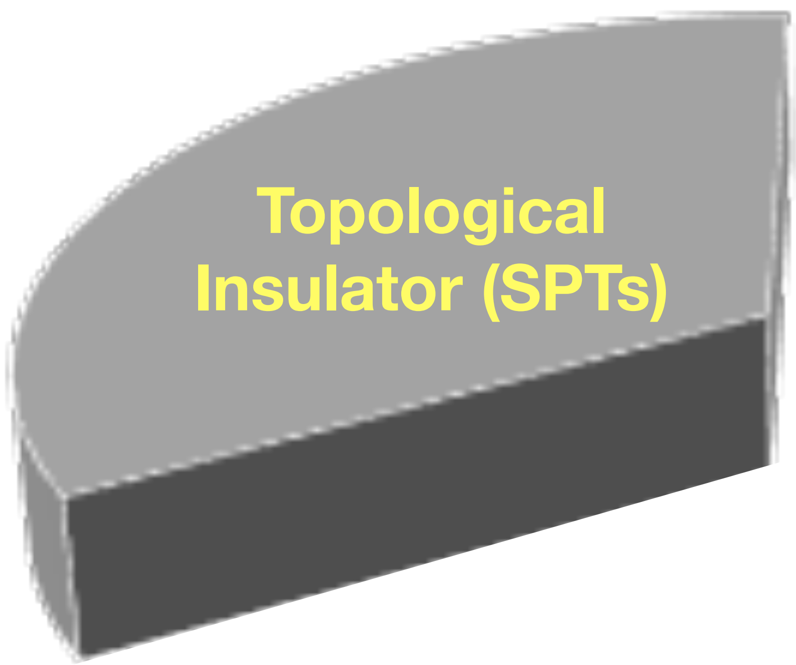
Zeljkovic lab

<u>Very different from STM</u>

Local information of 2 operator expectation value.

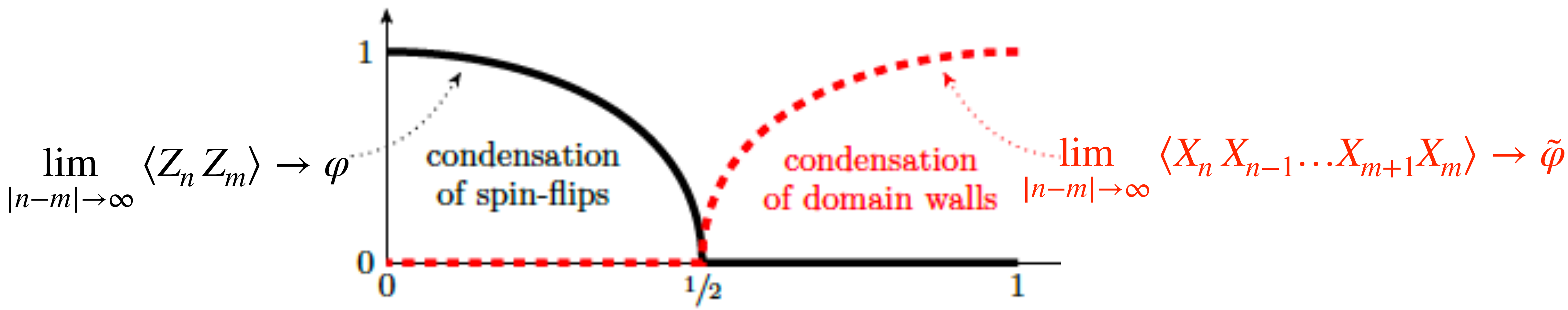
$$\mathcal{N}(x, E) = \int dt\, e^{-i\omega t} \langle c_x^\dagger(t) c_x(0) \rangle$$

Part 2:



- String orders and 1D Symmetry Protected Topological Insulators (SPTs).
  - Decorated domain wall construction of SPTs.
  - String order parameter from decorated domain walls
  - Experiments

Symmetry Protected Topological Phase

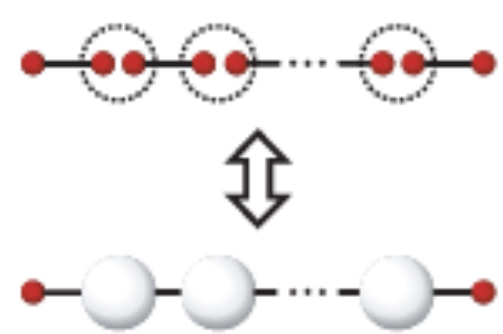


Ising Model - only two phases:

1. Symmetric Phase (disorder)
2. Broken Symmetry (Order)

With *more* symmetry, one can have further distinctions between *disordered* phases.

Famous Example - AKLT Spin - 1 Chain

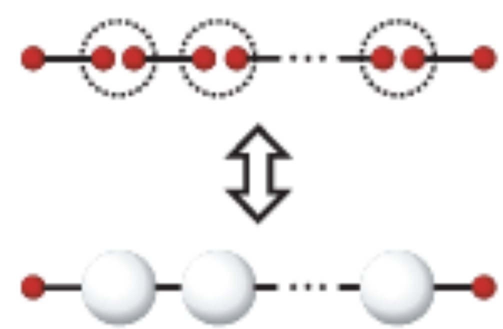


Kaltenbaek `10

Spin-1 Affleck Kennedy Lieb Tasaki Chain

With *more* symmetry, one can have further distinctions between *disordered* phases.

Famous Example - AKLT/Haldane Spin - 1 Chain



Kaltenbaek `10

$$H = J \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1}$$

$$|\Psi\rangle = |0,+,-,0,0,0,+,-,+,0,-\ldots\rangle + \ldots$$

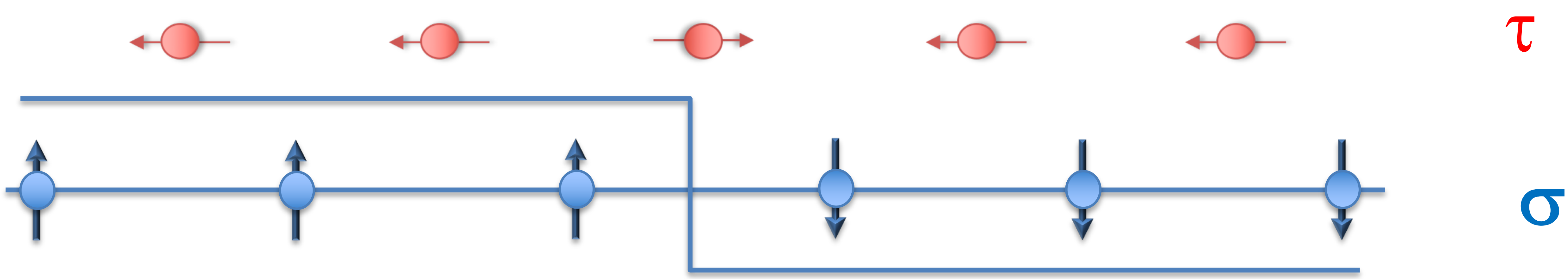
$$|+,-,+,-,+,-\ldots\rangle + \ldots$$

- Disordered Phase, but with string order.

Experiment - edge states of Y_2BaNiO_5 (Takagi PRL `03)

Cold atoms: Browaeys and Bloch Groups 1810.13286 s/2103.10421

Condensate of Decorated Defects



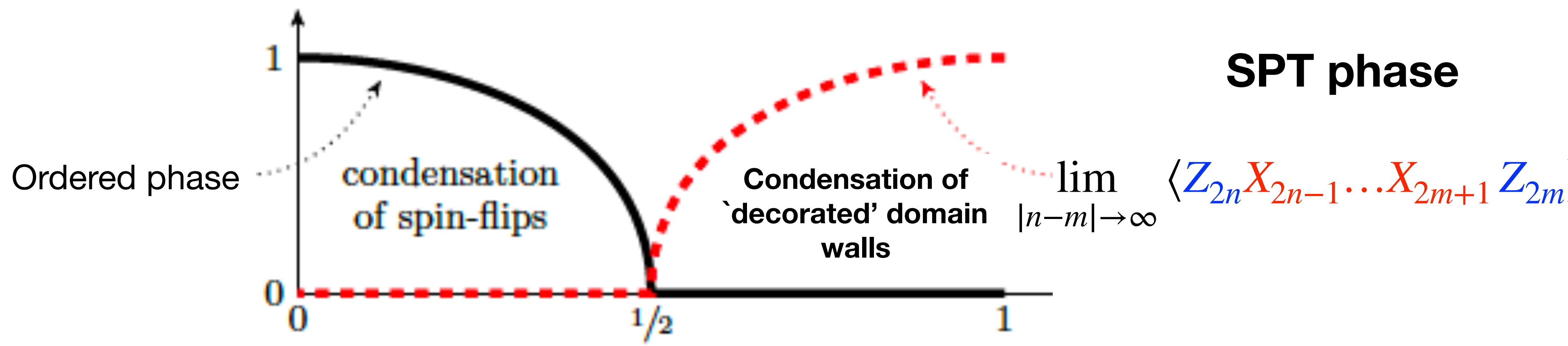
- 1D topological phase with G = $Z_2 x Z_2$ symmetry.
  - Two Ising models.

Condense domain walls of Z_2 with Z_2 charge.

$$H = -\sum_i \left(Z_{2i+1} X_{2i} Z_{2i-1} + Z_{2i} X_{2i-1} Z_{2i-2} \right)$$

i Exactly Soluble Hamiltonian (Cluster State) Son, Amico, Vedral

Condensing Domain Walls with “Charge”



$$H_i = -Z_{2i+1} X_{2i} Z_{2i-1}$$

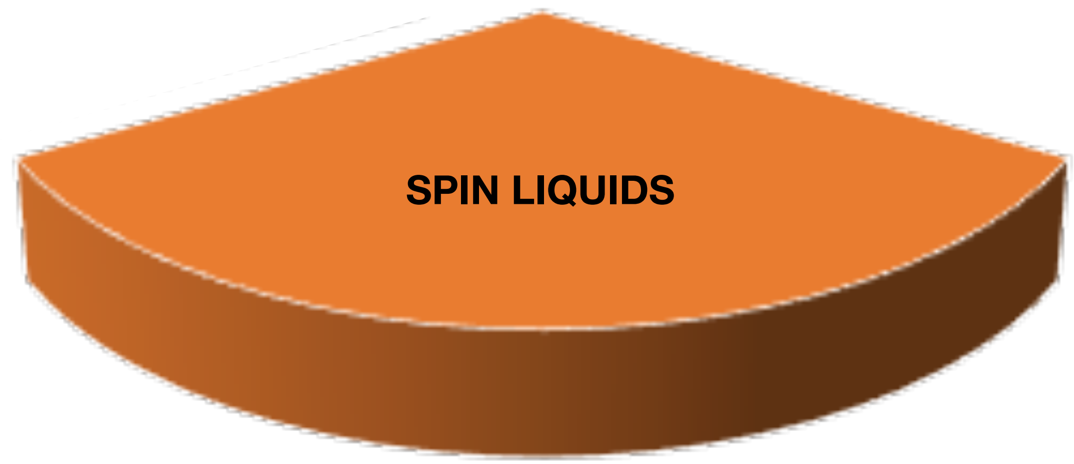
$$X = +1\,(\text{vacuum})\ \&\ X = -1\,(\text{spin}-\text{flip})$$

$$Z_{2i+1} Z_{2i-1} = -1 => \text{DomainWall}$$

- The Hamiltonian attaches `spin flips’ if there is a domain wall present.
- Condensate of domain walls `decorated’ by spin flips on opposite sublattice.
- A string order parameter? $\lim_{|n-m|\to\infty} \langle Z_{2n} X_{2n-1} \dots X_{2m+1} Z_{2m} \rangle$

Decorated domain walls - X. Chen, Y. Liu, AV . Nature Comm. `14

Part 3



- *Intrinsic* topological order & Quantum Spin Liquids
- Toric Code topological order
- A realizable quantum spin liquid - the `ruby lattice' Rydbergs.
- Nonlocal probes of intrinsic topological order.

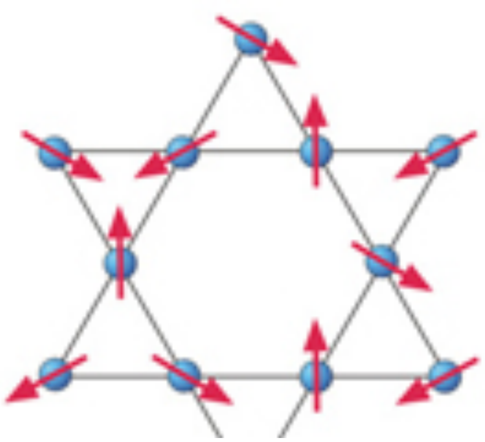
Beyond symmetry breaking - Quantum Spin Liquids

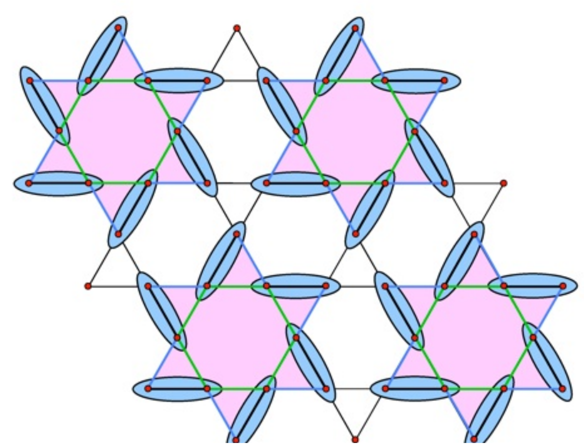
RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR ?*

P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

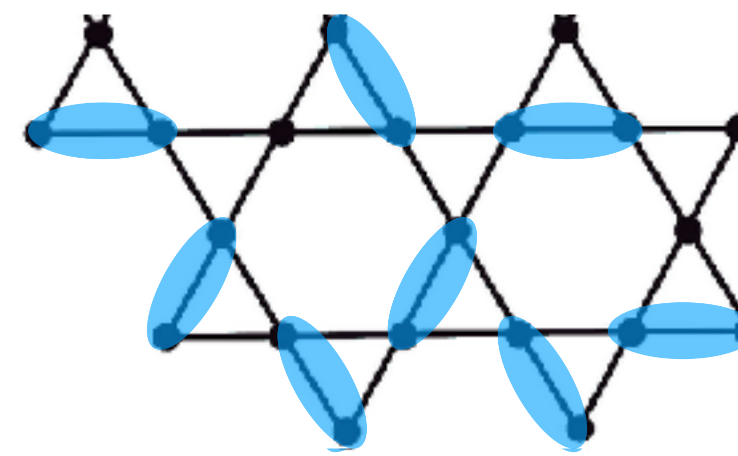
1973:

"Topological Order"



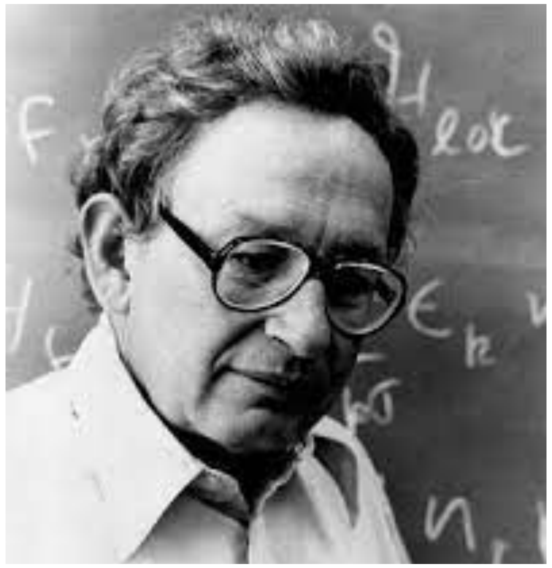


Valence Bond Crystal



Resonating Valence Bonds

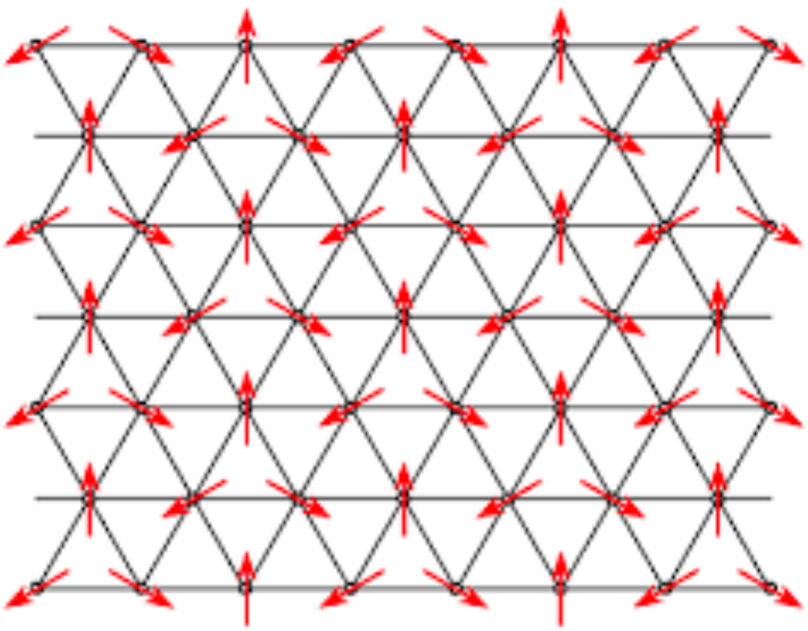
Topological order *beyond* FQHE

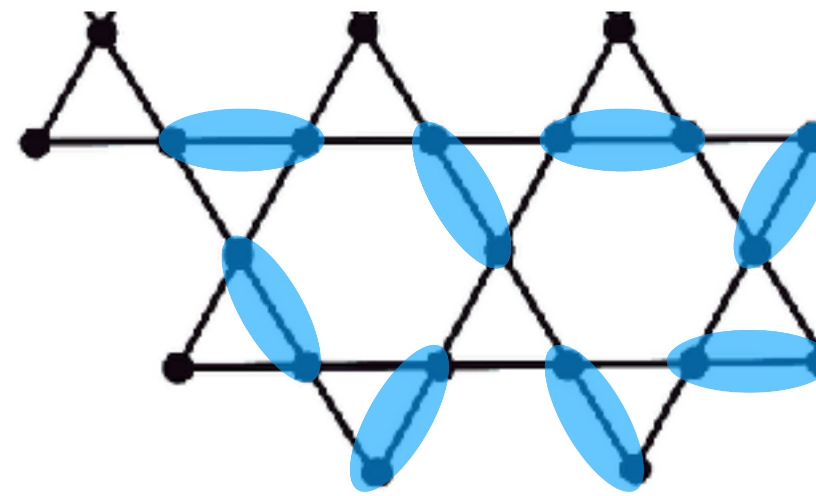


RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

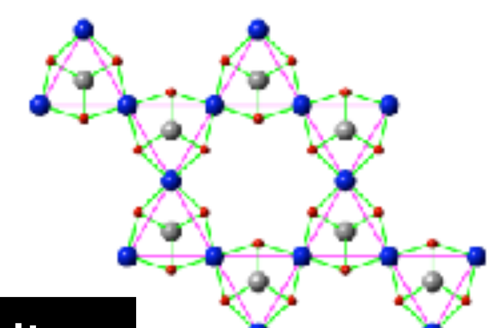
P. W. Anderson
Bell Laboratories, Murray Hill, New Jersey 07974
and
Cavendish Laboratory, Cambridge, England

1973:





(b) $ZnCu_3(OH)_6Cl_2$



Resonating Valence Bonds

BUT - Tendency to order

Herbertsmithite

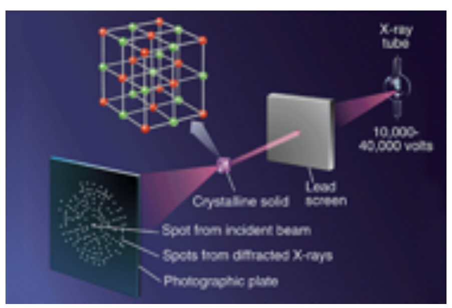
Experimental Progress- but *no* clearcut example

Classical Orders versus Topological orders

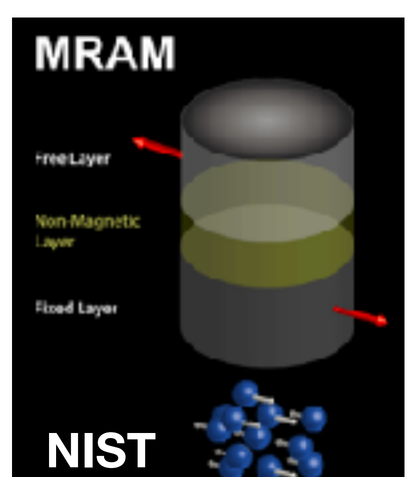
Landau Order: $\langle \varphi \rangle \neq 0$

(i) Measure order parameter experimentally to diagnose a phase:

Eg. X-rays on a Crystalline solid (Density(Q))



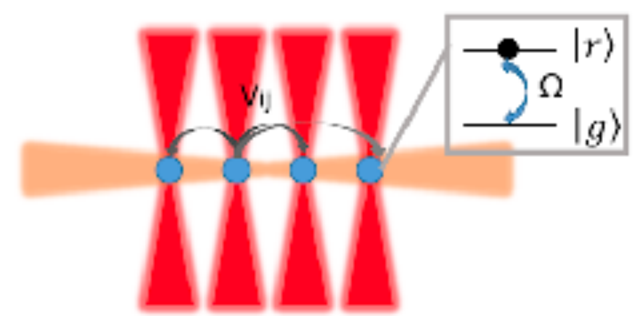
(ii) Many practical applications (eg. Magnetic memories):



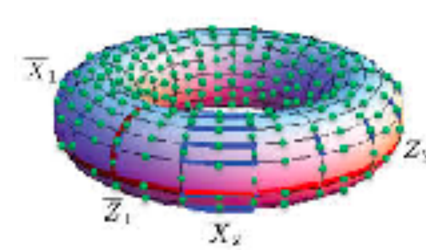
Topological Order:

- How to measure?

- New Platforms for realization?

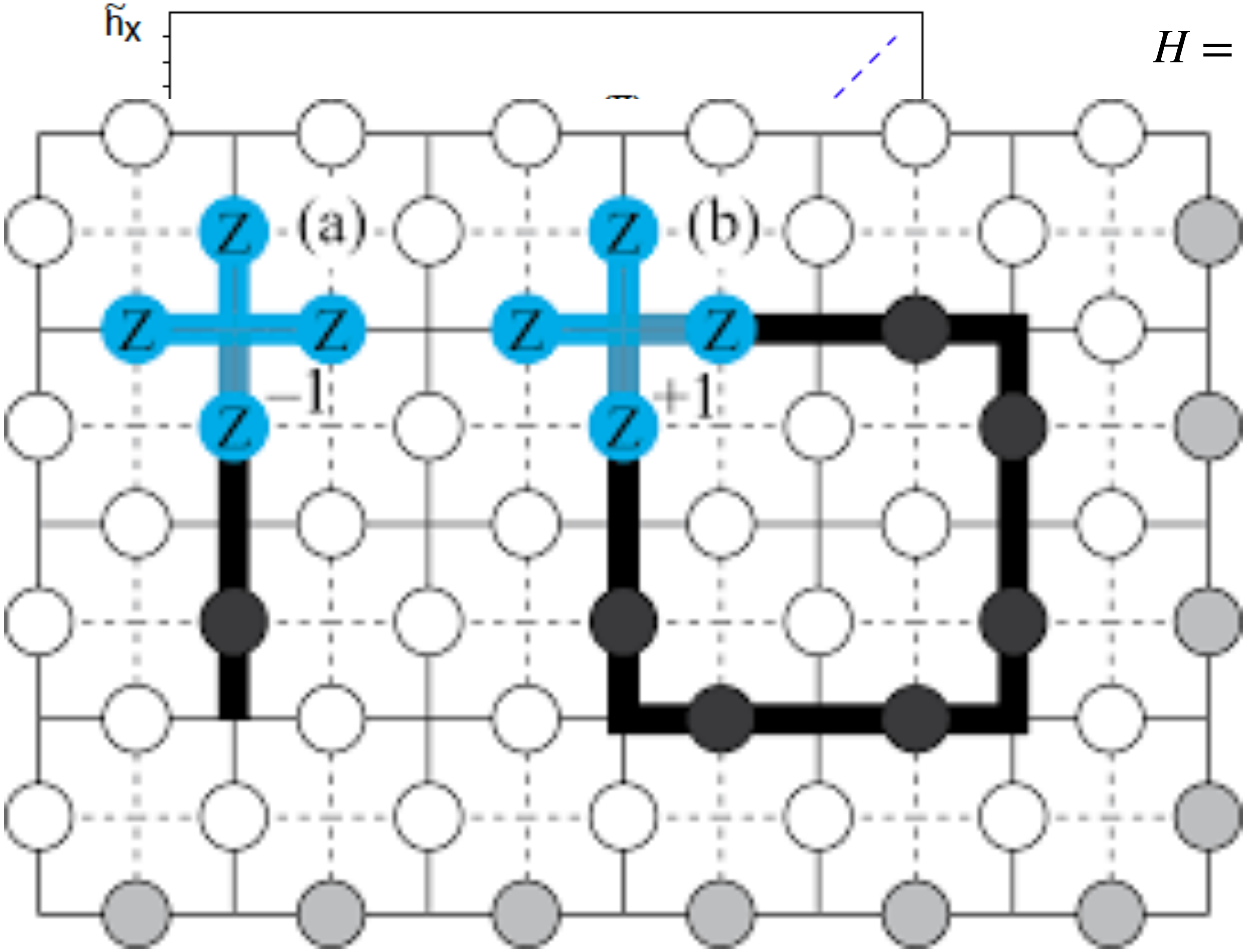


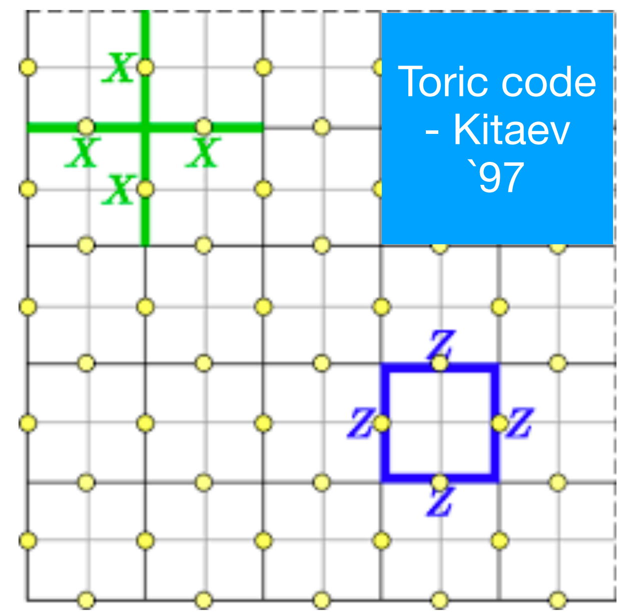
- quantum memories/computing



Toric Code

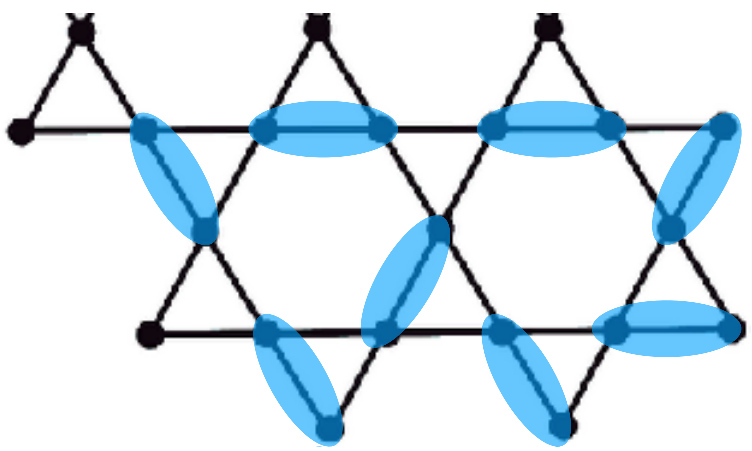
$$H = -\sum_p ZZZZ - \sum_s XXXX$$

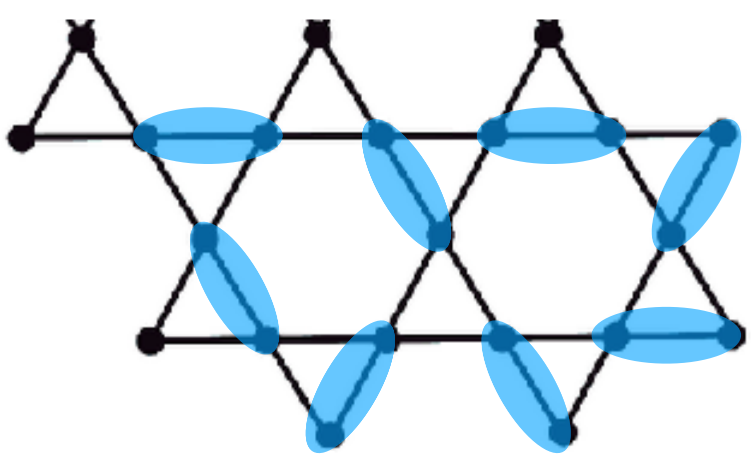




Ground state - superposition of closed loops.
Electric field lines “binary or Z_2” e= (0,1)

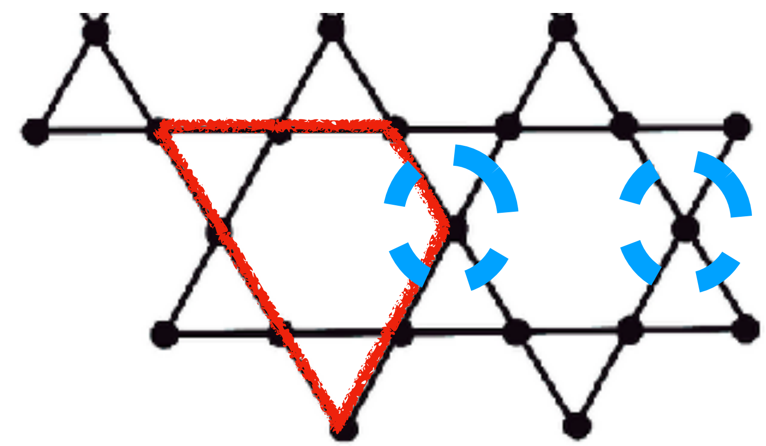
Condensates of Loops - Topological Order





Reference State

States represented by closed loops



$$\Omega = |\,\square\,\rangle + |\,{}^{\square}\;{}_{\square}\,\rangle + \ldots$$

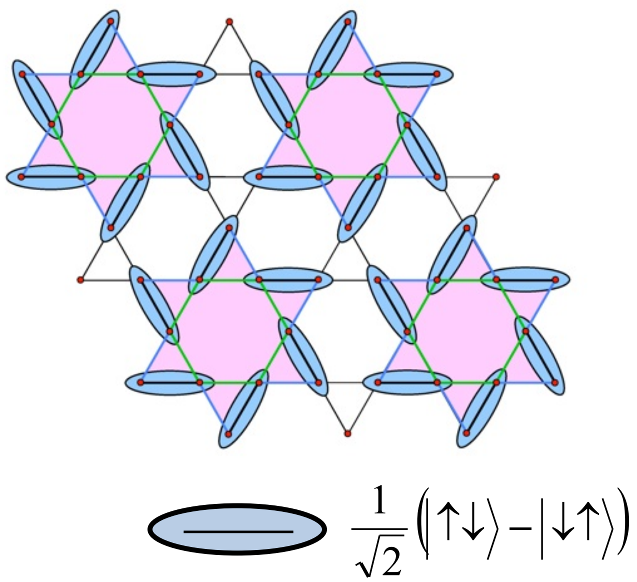
Deconfined phase of an *Emergent* Gauge theory.

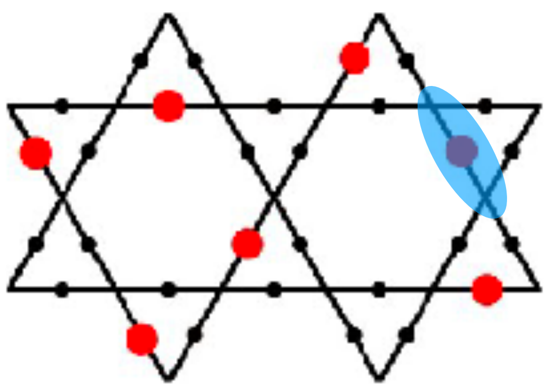
$$\nabla \cdot E = 0 \pmod 2$$

RVB/Spin Liquids in Rydberg Systems?

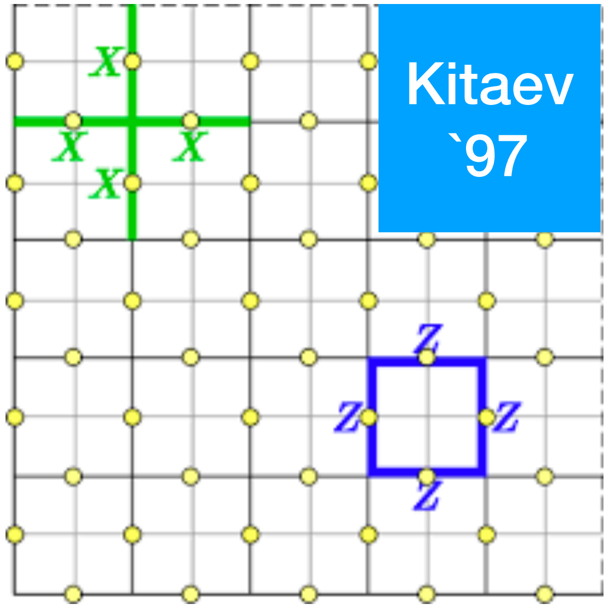
- Spin liquids where microscopic degrees of freedom are spins.

- Soluble model - the toric code





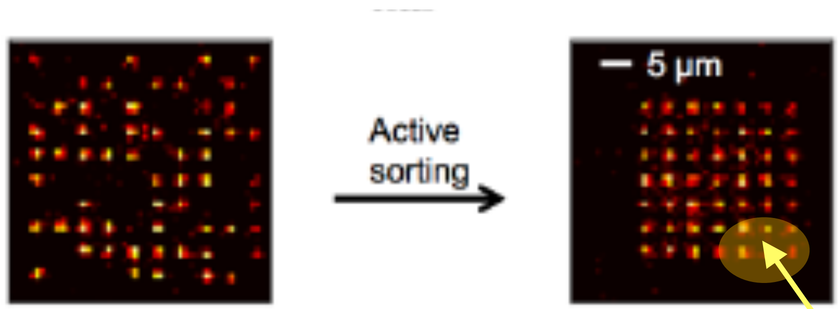
Allowed state



Toric code -
With 4 body interactions

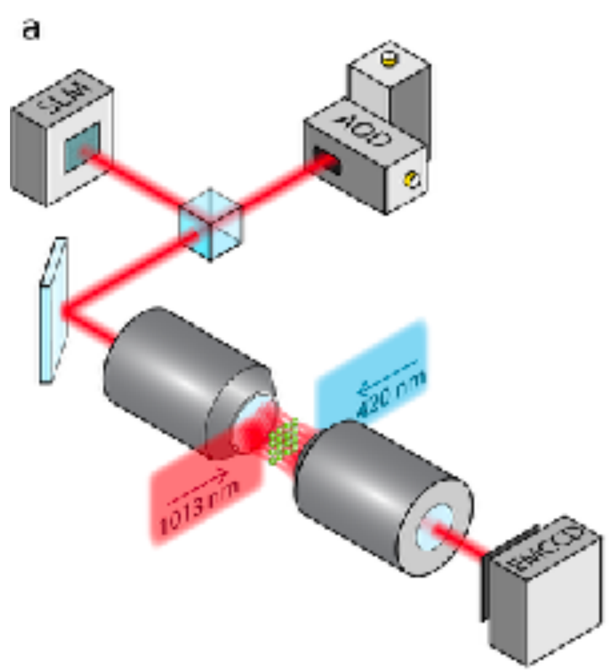
Wegner `71; Fradkin&Shenker `79
Rokhsar and Kivelson `88; Read&Sachdev `90; Sachdev `92; Wen `91
Moessner and Sondhi; Misguich et al. 2000. Balents Fisher Girvin `01; Motrunich Senthil `02

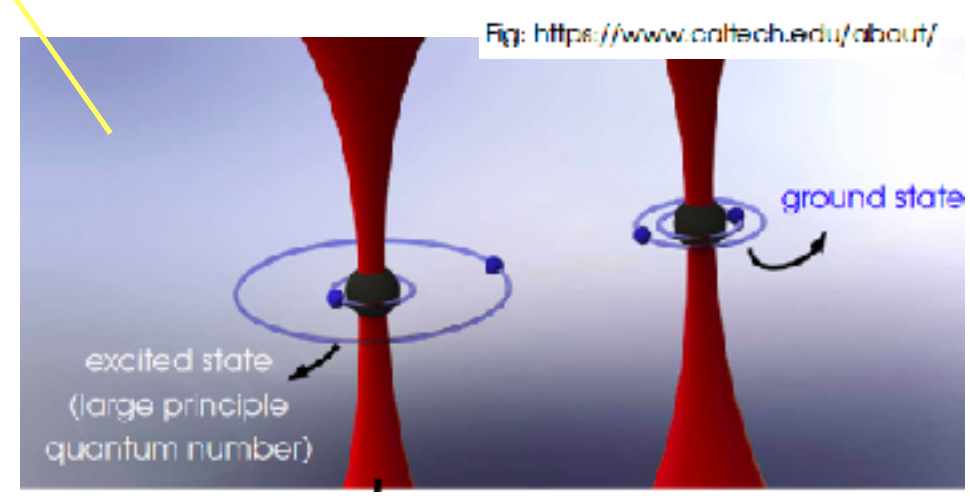
Rydberg atom array

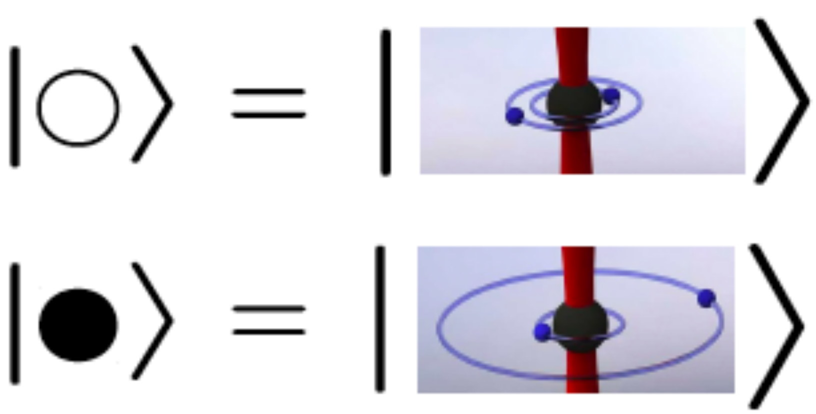


Quantum number ~70 of Rb atom - large van-der-Waals interactions.
Tunable lattices

Ebadi et al. arXiv:2012.12281
Scholl et al. arXiv:2012.12268

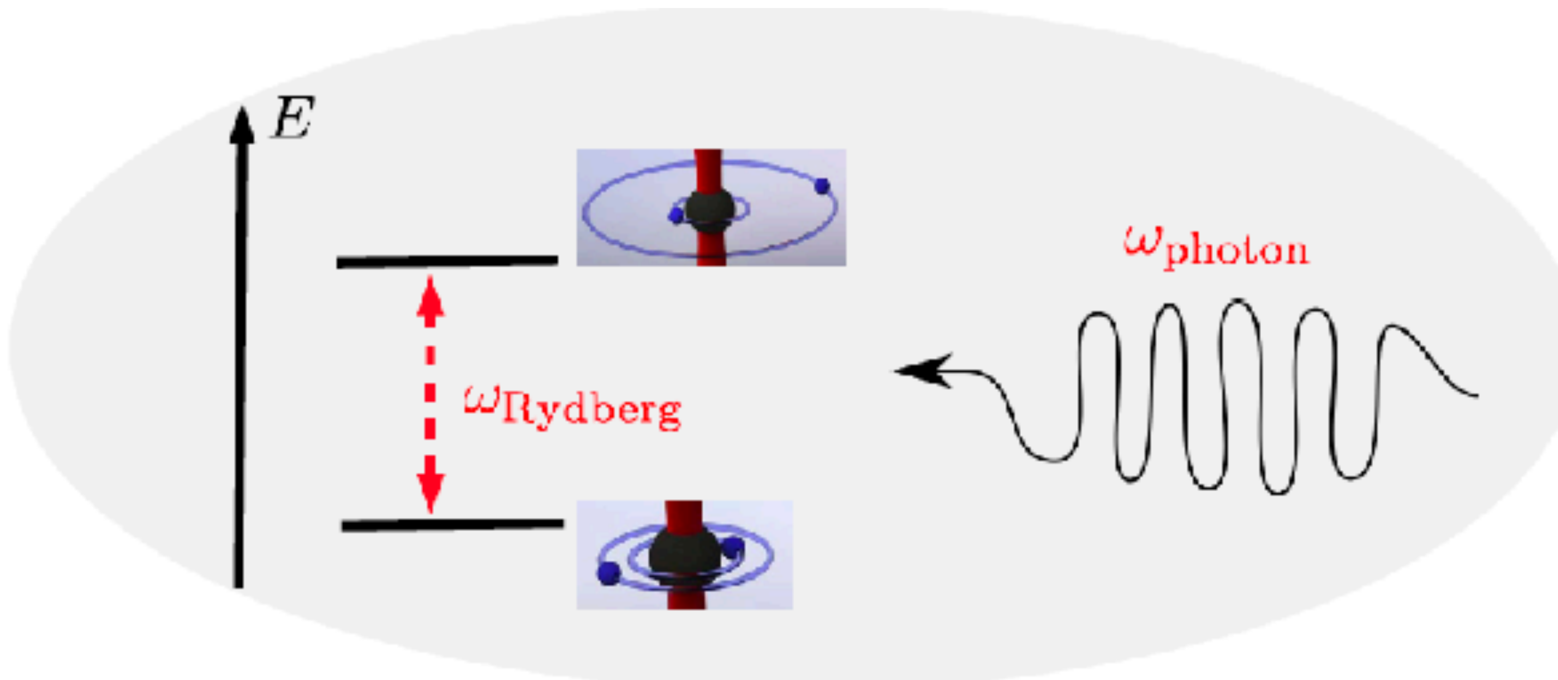






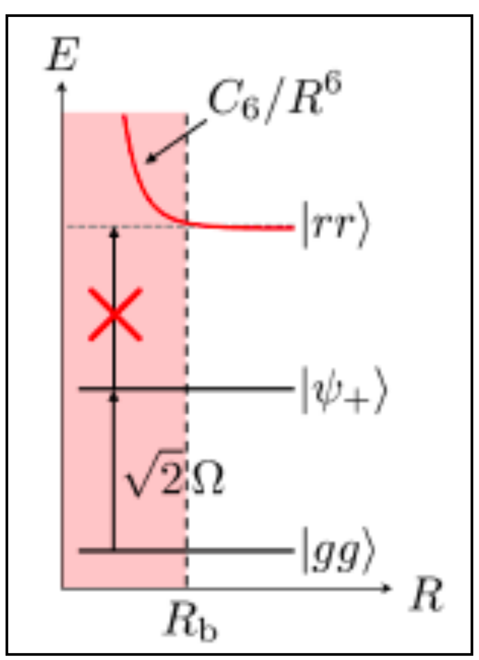
Quantum Bit

Rydberg Hamiltonian

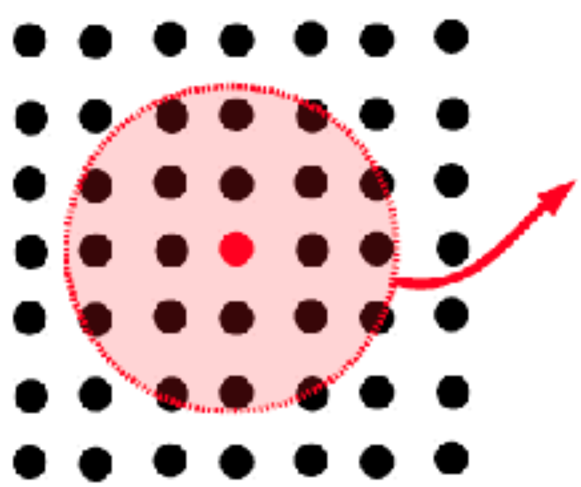


Described by $H = \frac{\Omega}{2}\sigma^x - \frac{\delta}{2}\sigma^z$ with

- Ω = Rabi frequency
- $\delta = \omega_{\mathrm{photon}} - \omega_{\mathrm{Rydberg}}$ = "detuning"



Browaeys and Lahay



$R_b \approx 2a$

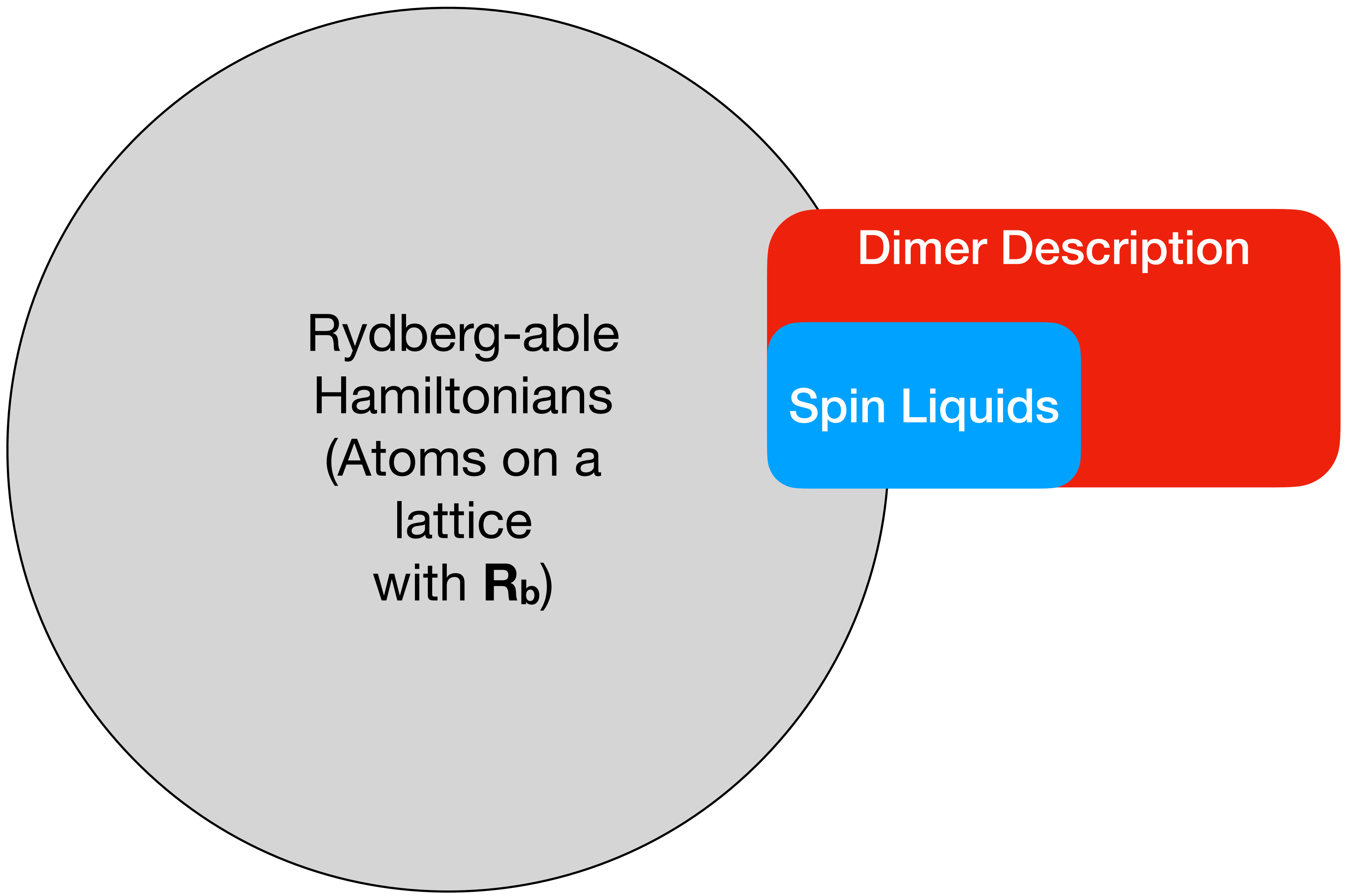
$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$

projects out double-occupation

(Sachdev-Sengupta-Fendley)

Jaksch et al. PRL (2000)
Lukin et al. PRL (2001)

Are Spin Liquids Rydberg-able?

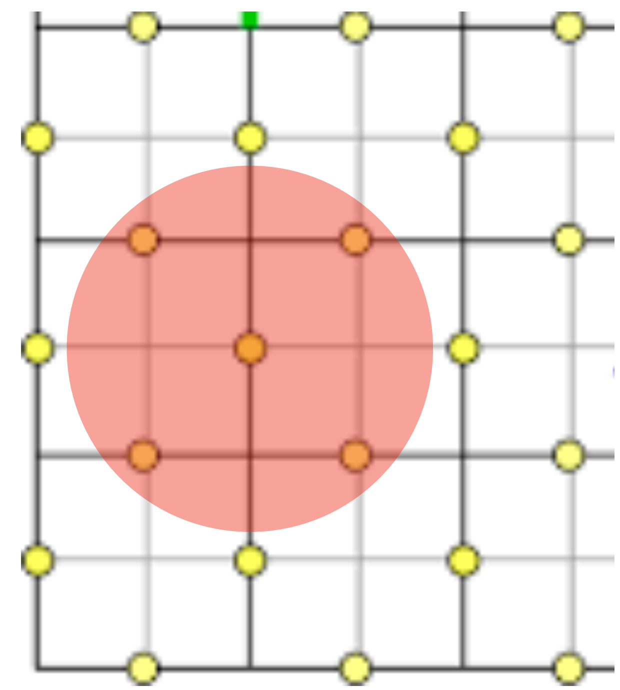


STEP 1: Does lattice geometry+ blockade radius support a Dimer-monomer Hilbert space?

STEP 2: Do quantum fluctuations from Ω lead to spin liquids?

Are Spin Liquids Rydberg-able?

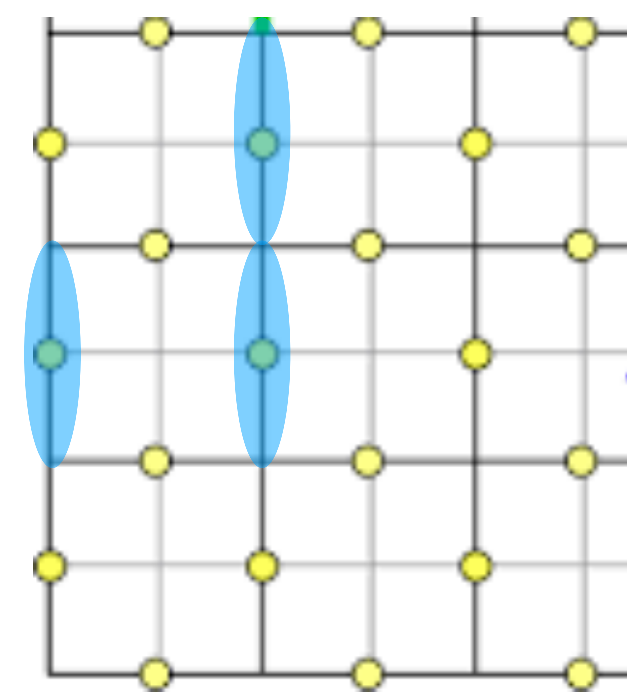
STEP 1: Does lattice geometry+ blockade radius support a Dimer-monomer Hilbert space?



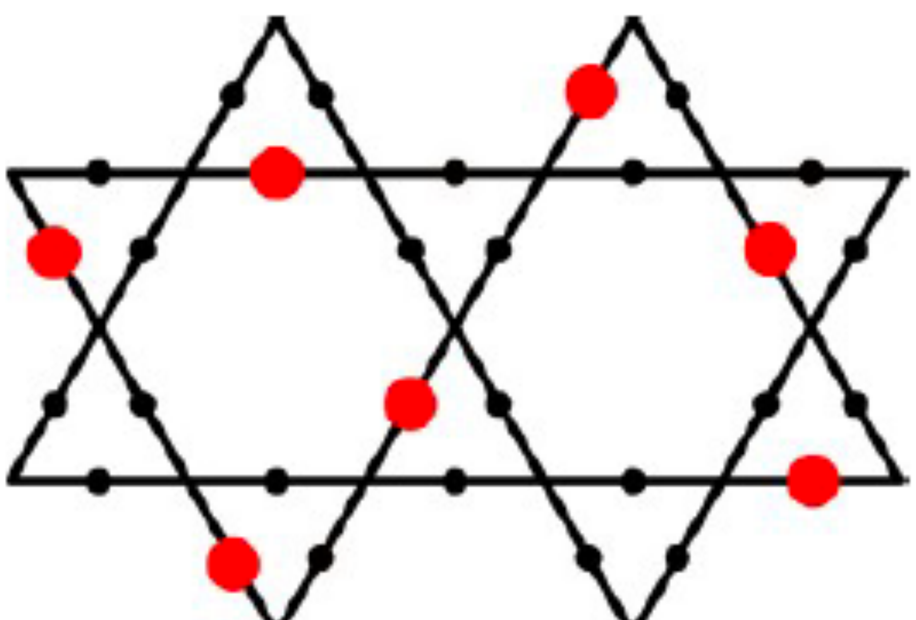
$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta\sum_i n_i$$

projects out double-occupation

Within radius R_b



Dimer Model From Blockade

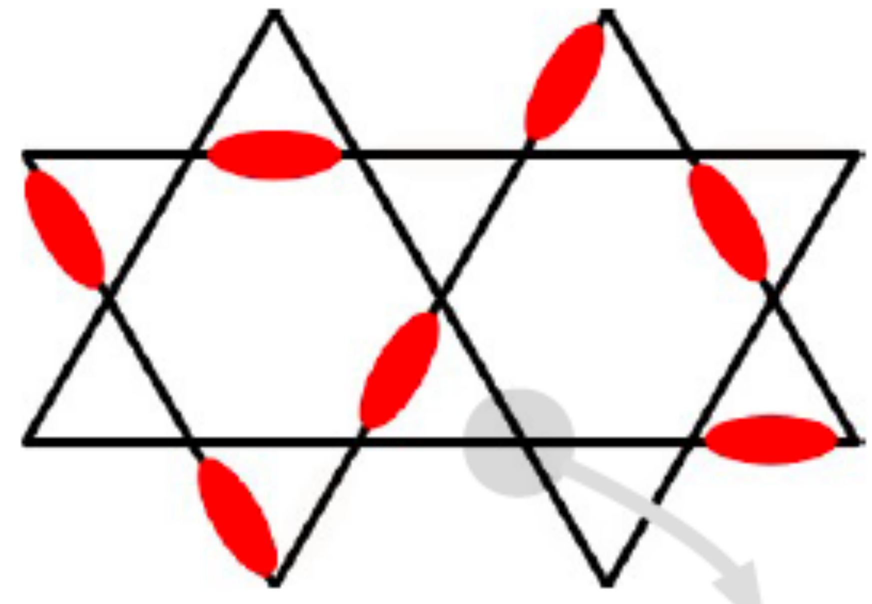


Links of the Kagome

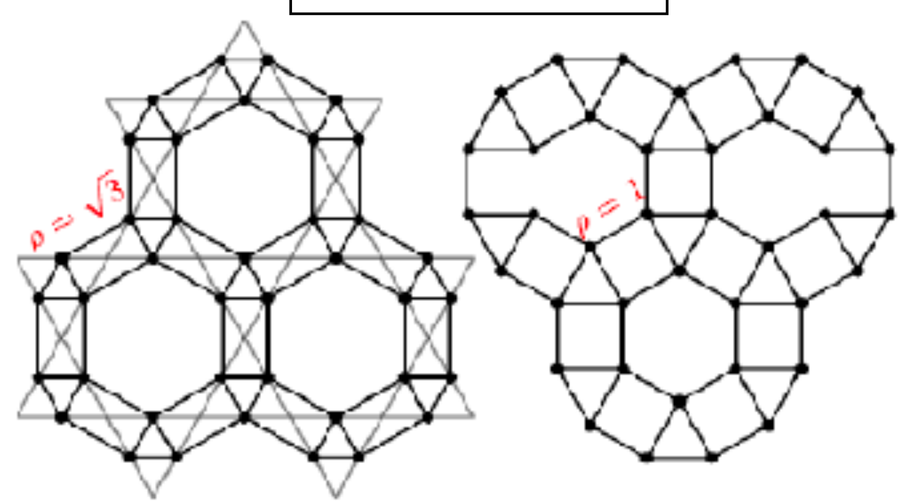
$$R_b \leq 2a$$

$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$

projects out double-occupation



RUBY



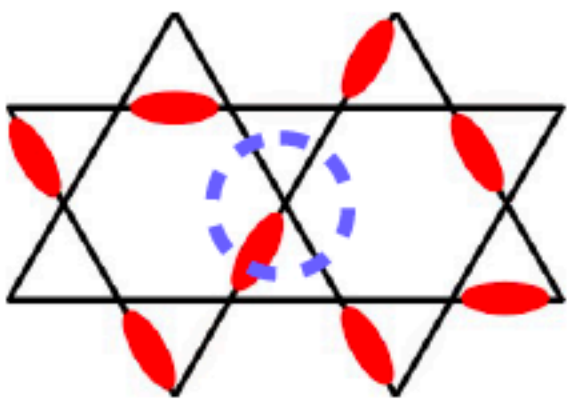
Step 1: Ruby Lattice -> Dimer Hilbert space

Rydbergs on other lattices: Glaetzle et al. PRX (2014); Samajdar et al., PNAS (2021)

Z_2 Spin Liquid?

Step 1: Ruby Lattice -> Dimer Hilbert space

The hardcore dimer constraint = Gauss law $\nabla \cdot E = 1 \mod 2$

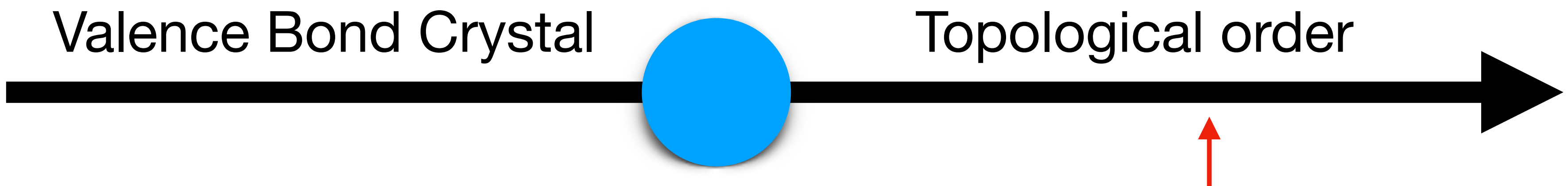


The parity around any vertex = -1

- *Dynamics* - confinement/deconfinement?

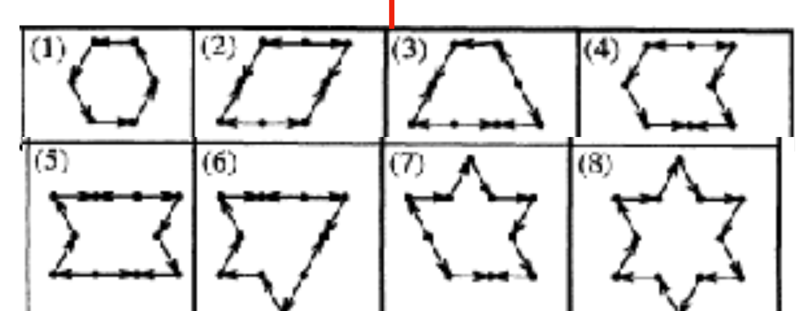
Confined -
Valence Bond Crystal

Deconfined -
Topological order



Dimer resonance

<u>Exact Solution:</u>
32 resonance terms with equal weight
Misguich, Serben, Pasquier '01
Zeng & Elser '95



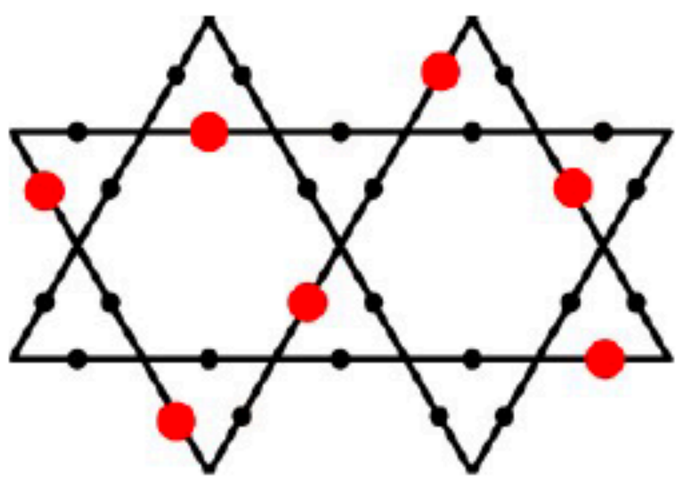
Towards Z_2 Spin Liquids in Rydberg Matter

Blockade model for atoms on links of kagome lattice

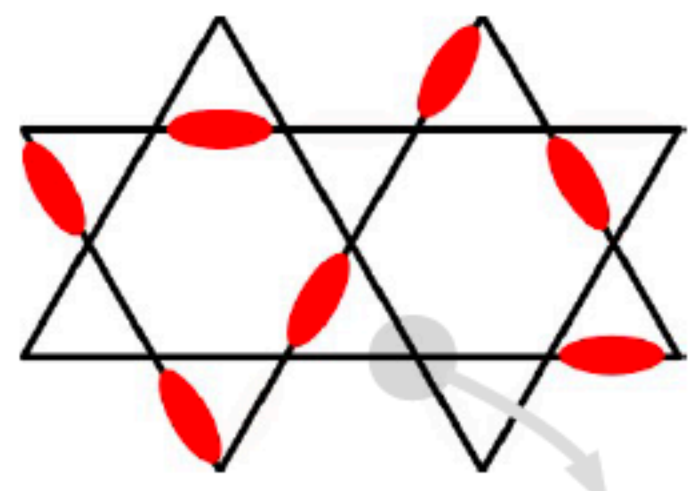
Ruby Lattice

$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta\sum_i n_i$$

projects out double-occupation $R_b \leq 2a$
in shaded red disk on left



=



Dimer state with one monomer

Dimer models from Rydbergs on other lattices: Glaetzle et al. PRX (2014); Samajdar et al., PNAS (2021)

Step 1: Ruby Lattice -> Dimer Hilbert space

Step 2: Phase Diagram of Ruby Lattice Model

$$H = \frac{\Omega}{2}\sum_i P\sigma_i^x P - \delta \sum_i n_i$$

Ruby Lattice

$R_b \leq 2a$

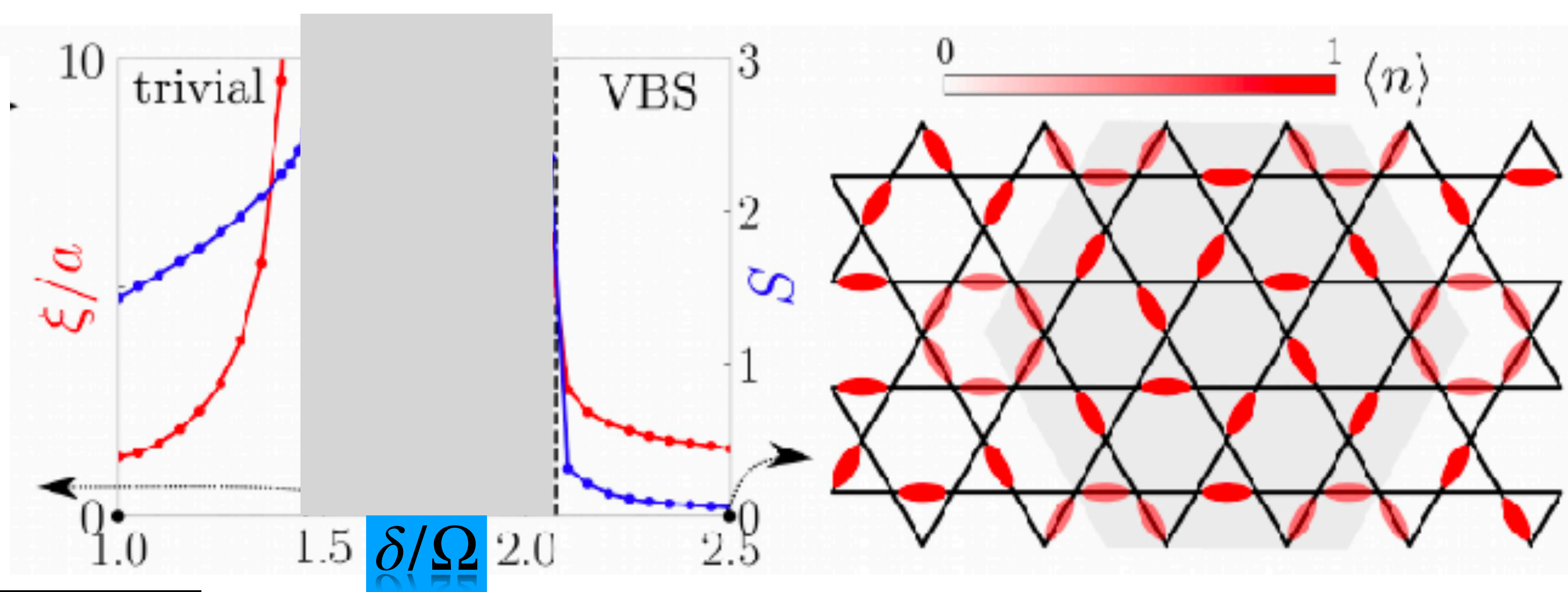
we put the model on an infinitely-long cylinder

→ use density matrix renormalization group (DMRG)

(White '92, Stoudenmire '13, Hauschild '18)

Ruben Verresen

Harvard



intermediate featureless phase

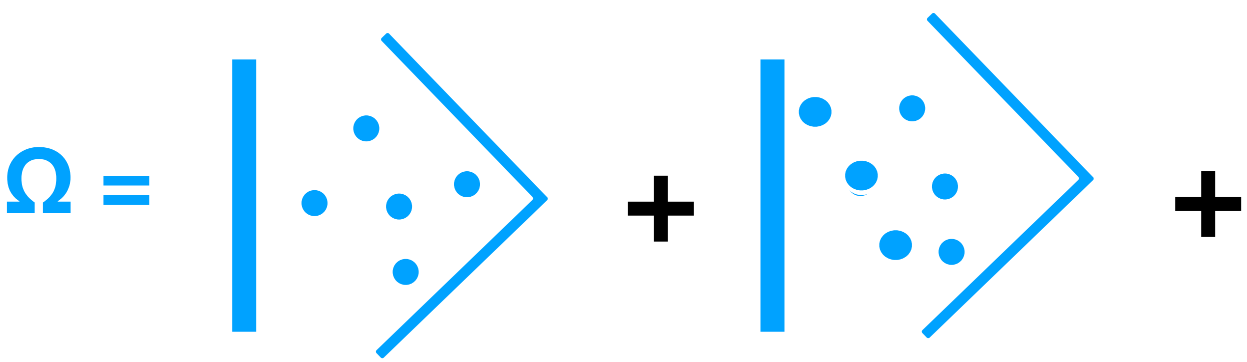
Ruben Verresen, Lukin, AV. PRX `21. arxiv 2011.12310

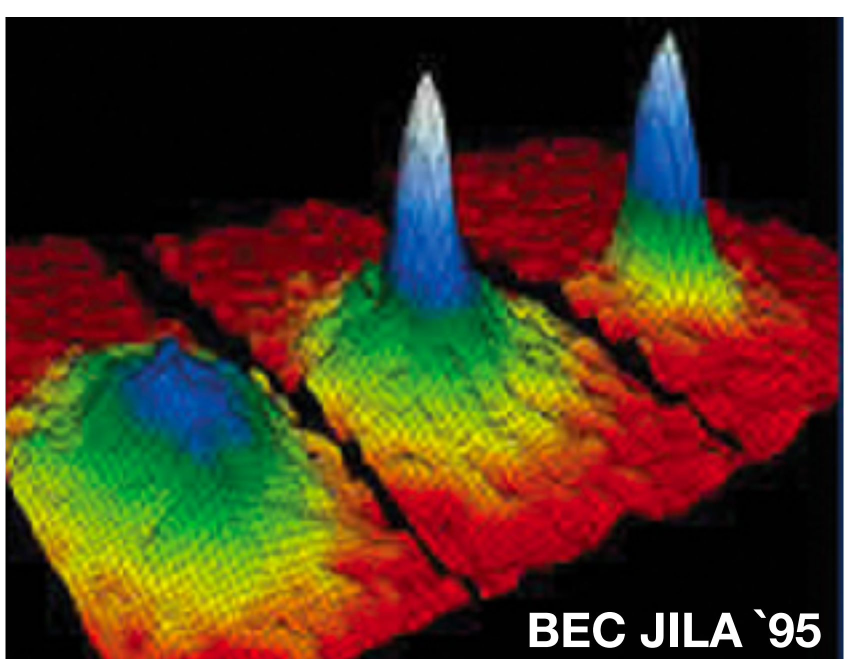
Particle Condensates - versus- Loop Condensates

Landau Order Parameter:

$$\langle \varphi \rangle \neq 0$$

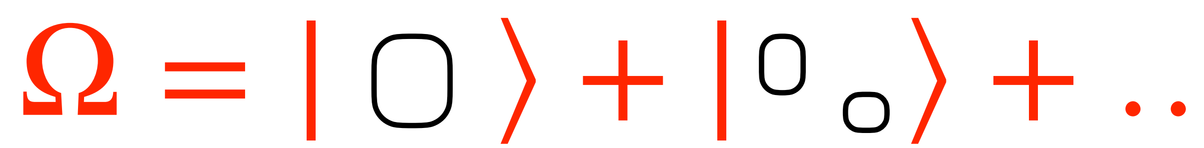
Condensate of particles:

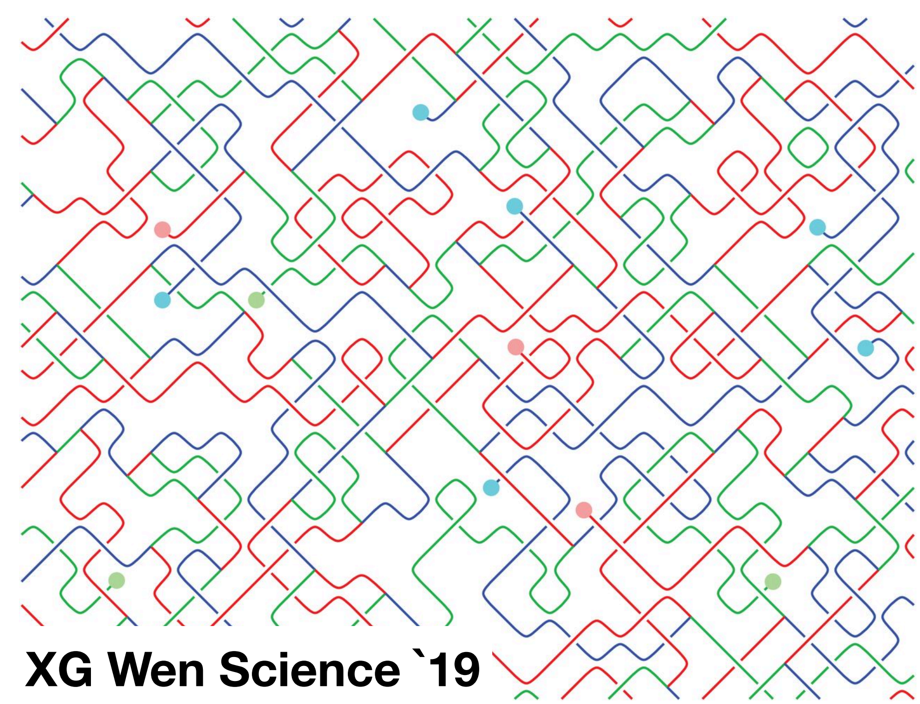




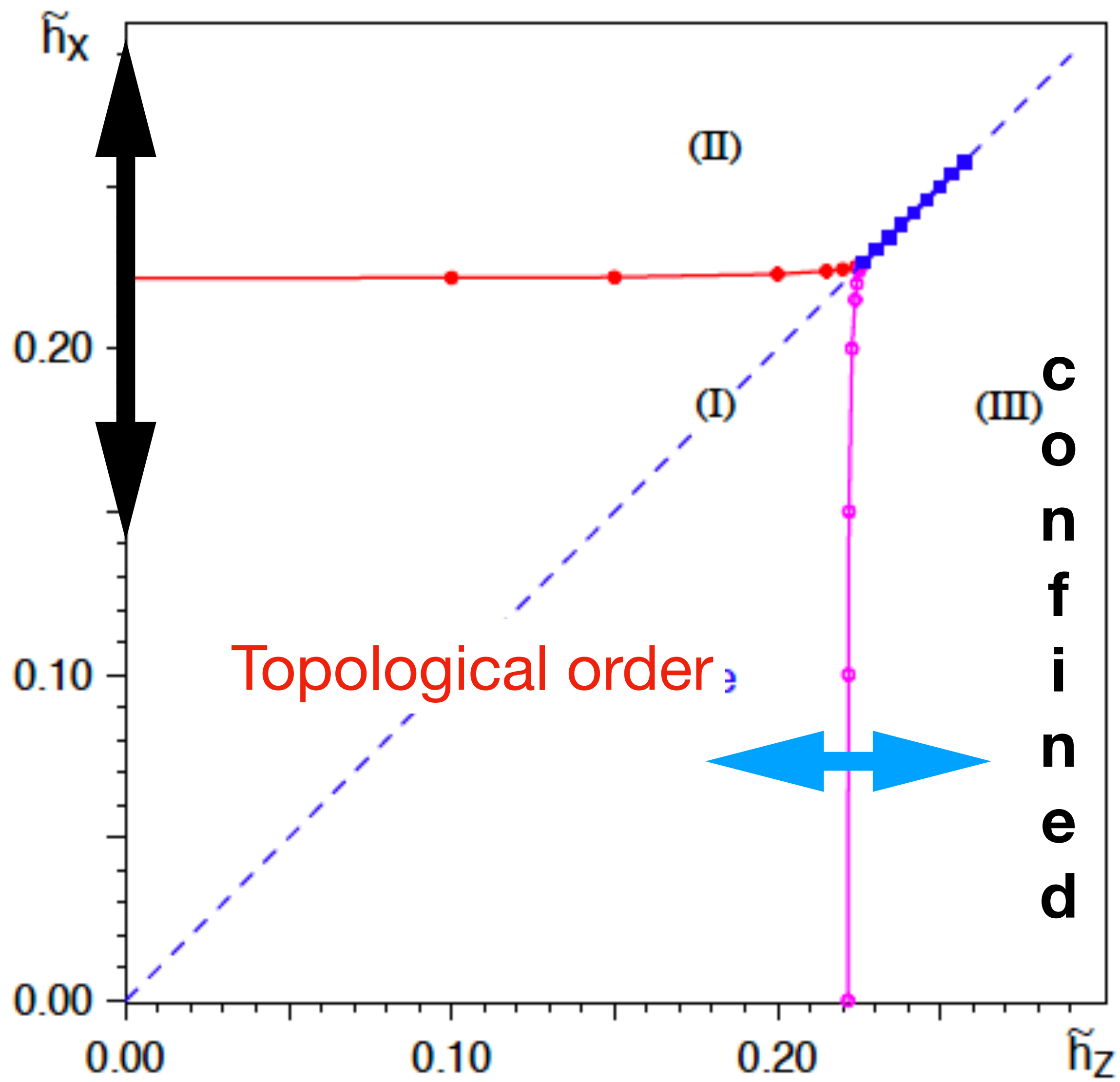
Topological order:

Condensate of closed loops:

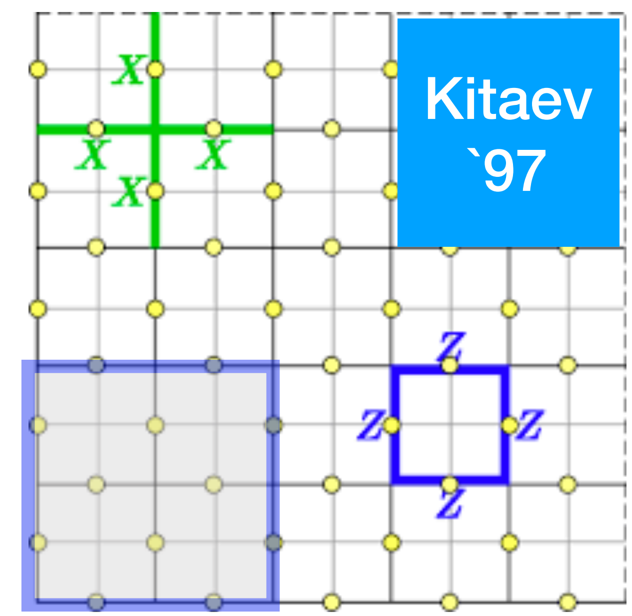




Wilson loop - Toric Code



$$H = -\sum_p ZZZZ - \sum_s XXXX - \tilde{h}_x \sum X - \tilde{h}_z \sum_i Z_i$$



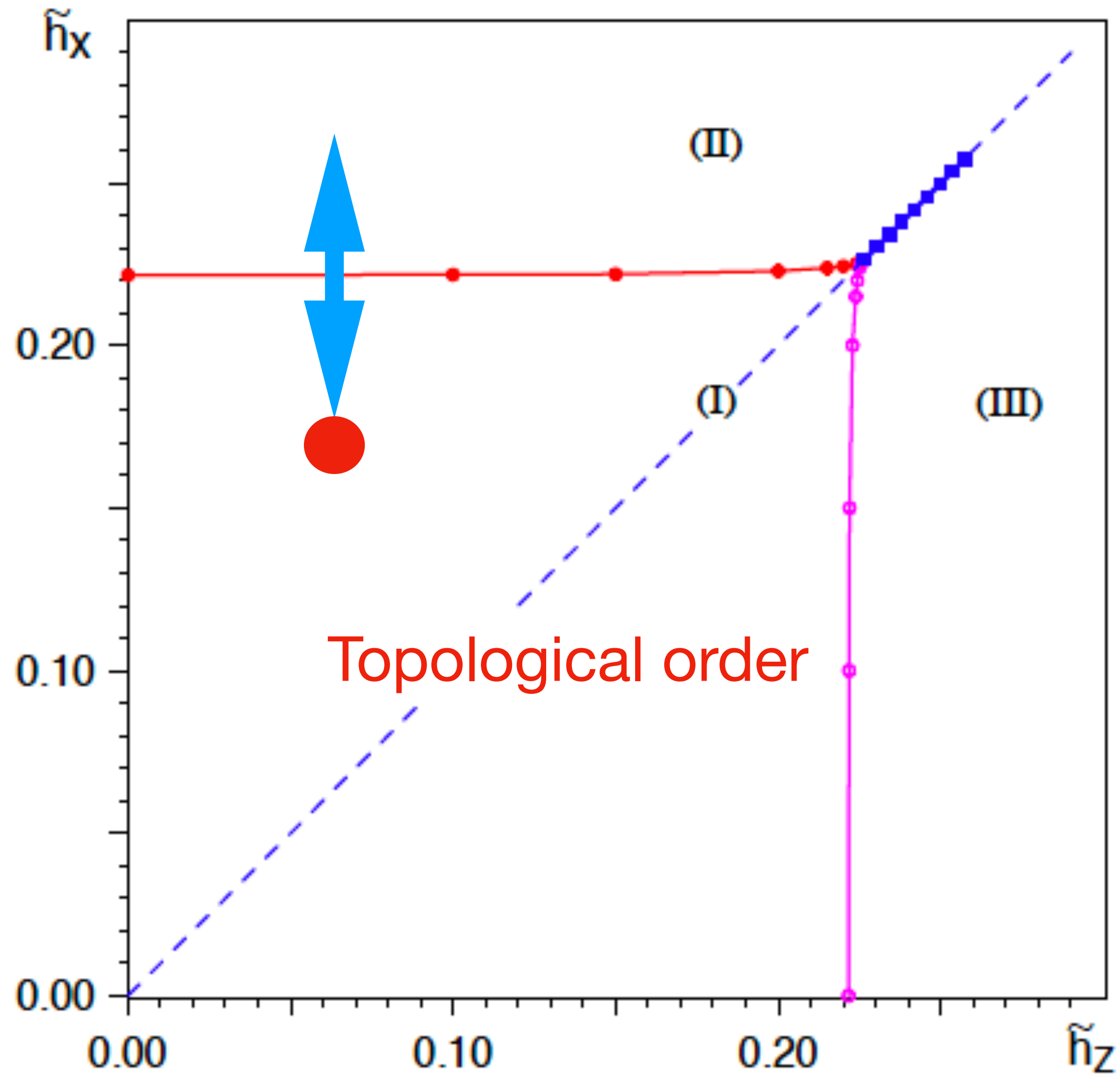
$W = \langle \prod Z \rangle$

How to distinguish?

`pure' gauge theory - Wilson loop: $W = \langle \prod Z \rangle$

$W \approx e^{-\text{perimeter}}$ $W \approx e^{-\text{area}}$ h_x

Generic Model - How to distinguish?



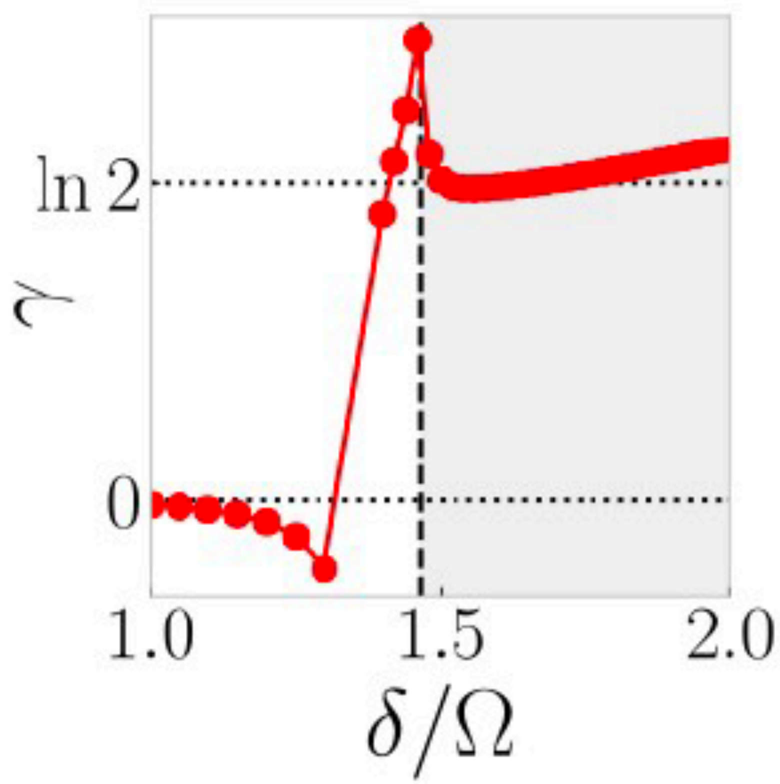
Wilson loop always perimeter law (with dynamical “matter”)

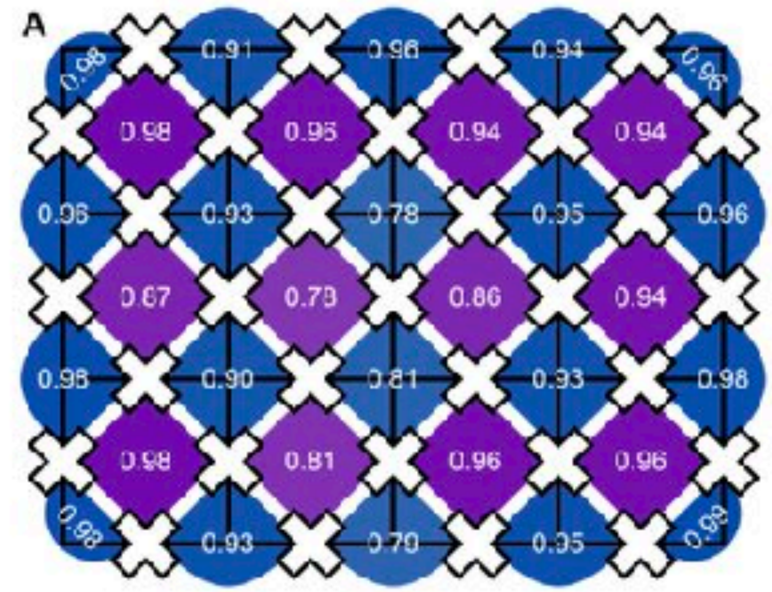
Ground state degeneracy (Witten)

Topological Entanglement entropy (Levin&Wen, Preskill&Kitaev)

Anyons & Modular Matrices (Wen; Zhang, Turner, Grover, Oshikawa, AV)

Generic Model - How to distinguish?





Satzinger et al. `21
Google's Toric Code

Topological Entanglement entropy (Levin&Wen, Preskill&Kitaev):

HERE -

NON-Local Order Parameter: open versus closed loops (FM order parameter)

ASIDE: entanglement entropy $e^{-S_2} = \mathrm{Tr}\left[\rho\rho\right] = \langle\rho\rangle$ *observable*: $\mathrm{Tr}\left[O\rho\right] = \langle O\rangle$

Particle versus Loop Condensate

→ a Bose-Einstein condensate (BEC) of particles:

$$\lim_{|i-j|\to\infty} \langle b_i^\dagger b_j \rangle \neq 0$$

only non-trivial if $b^\dagger$ is charged under U(1) symmetry

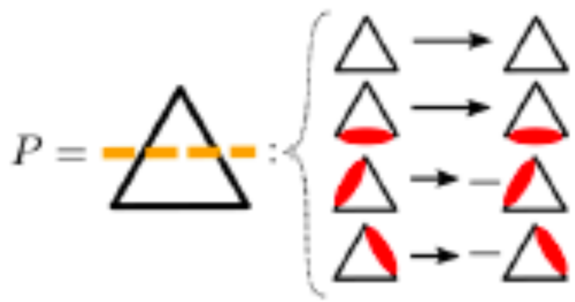
→ a Bose-Einstein condensate (BEC) of loops:

$$\left\langle \prod_{i\in \mathrm{loop}} U_i \right\rangle \neq 0$$

only non-trivial if $\langle \quad \rangle = 0$

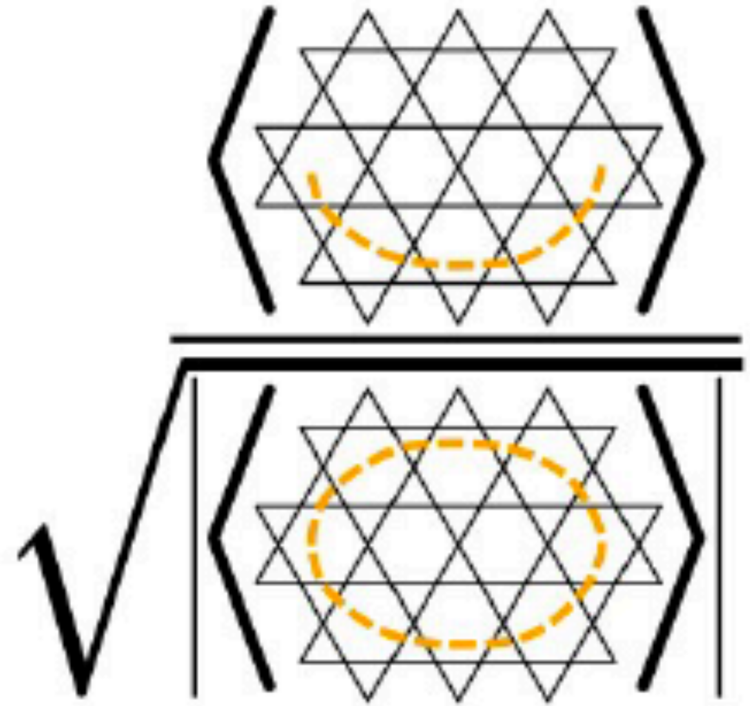
→ property of closed loops
→ open strings host anyons!

Fredenhagen-Marcu Order Parameter



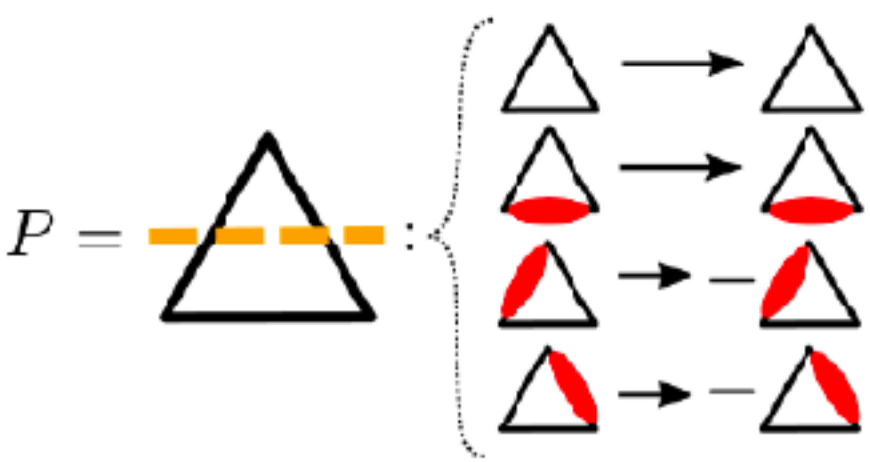
P - counts dimers

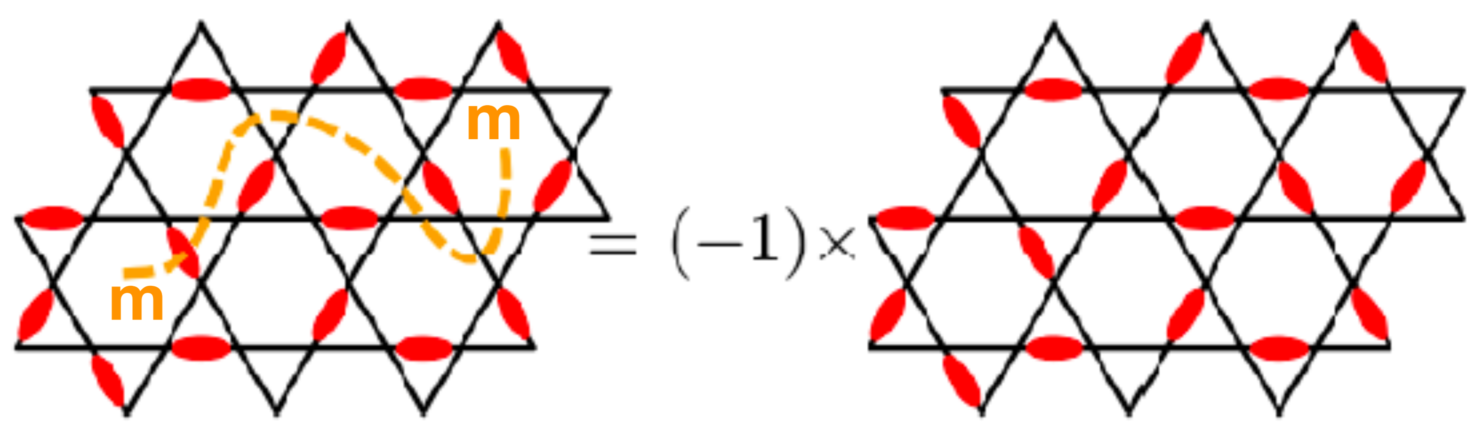
Fredenhagen-Marcu order parameter:



"$\prod \sigma^z$"

line operator $e^{i\pi \int E}$ is diagonal in the dimer basis

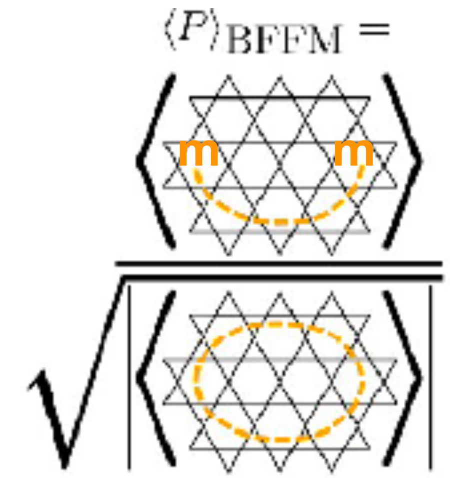




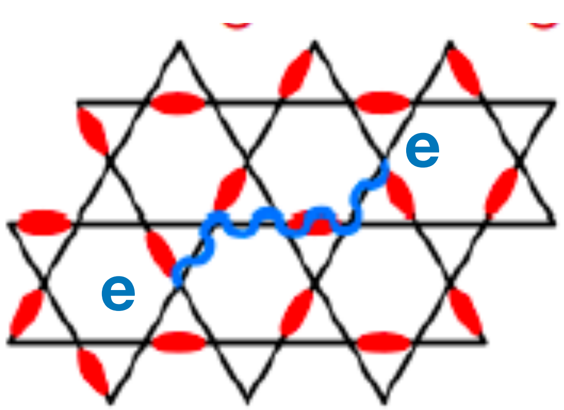
Ruben Verresen, Lukin, AV. PRX `21. arxiv 2011.12310

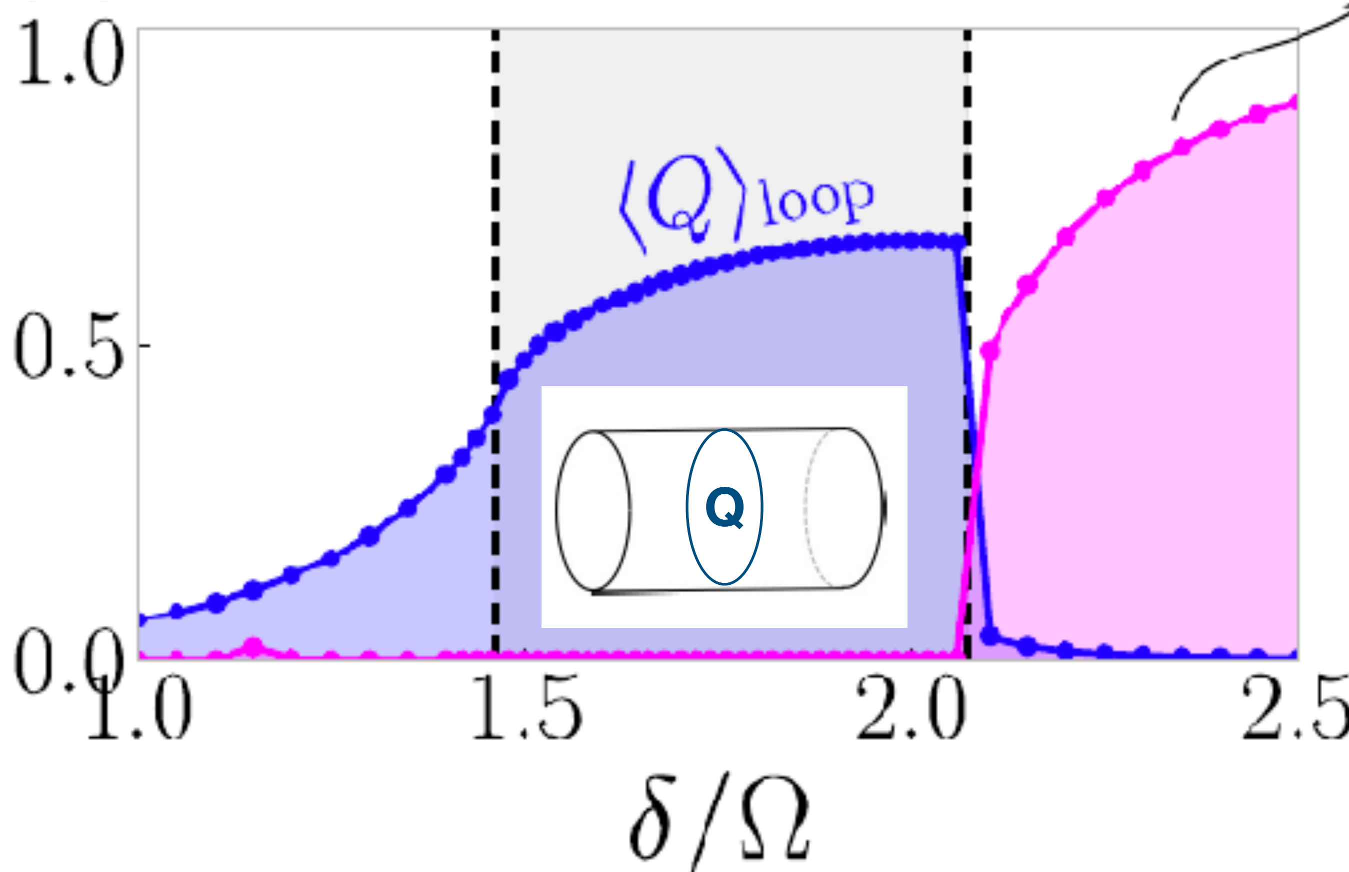
Diagnosing Phases

$\langle Q \rangle_{FM}$



$\langle P \rangle_{FM}$





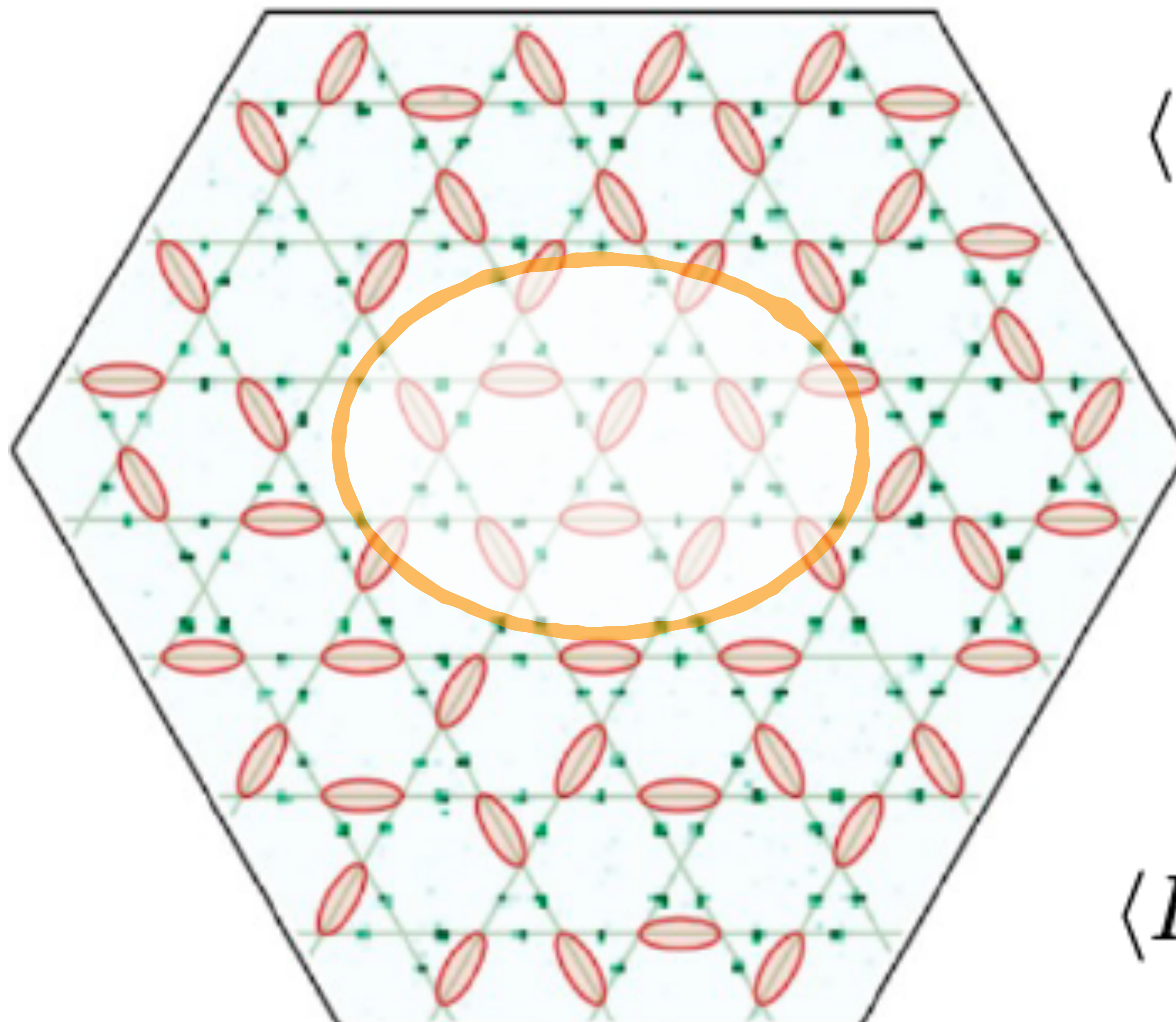
Towards an experimental realization

→ stability of spin liquid to van der Waals interactions

→ how to measure

Measuring Off-Diagonal String Operators

→ Diagonal string P measured from snapshots of atoms



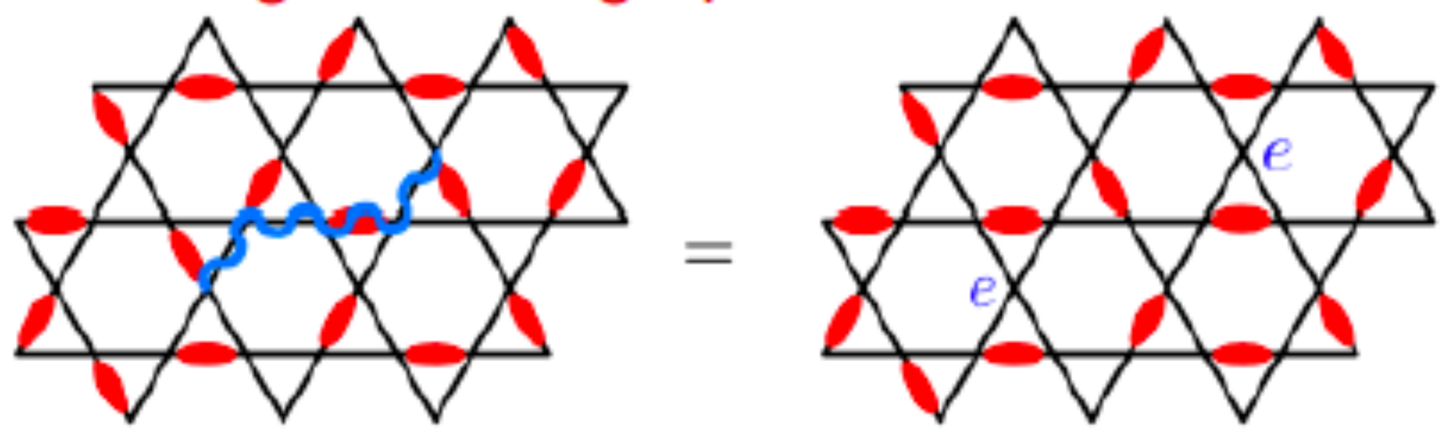
$\langle P \rangle$

$\langle P \rangle_{\mathrm{F}}$

Semeghini et al. Science `22. arxiv:2104.04119

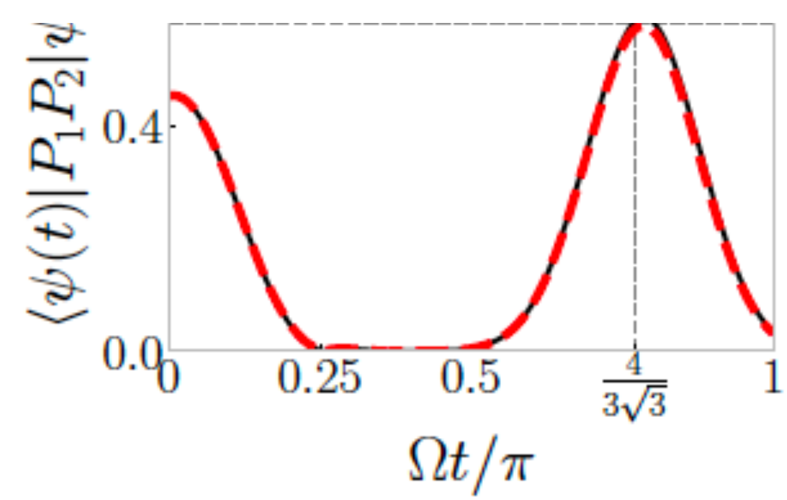
Fredenhagen-Marcu Order Parameter

To distinguish classical spin liquid from quantum spin liquid:
there is a dual off-diagonal string operator



Q - flips dimers

P - counts dimers




Ruben Verresen

We predicted a many body quantum quench can do the desired rotation!

Ruben Verresen, Lukin, AV. PRX `21. arxiv 2011.12310

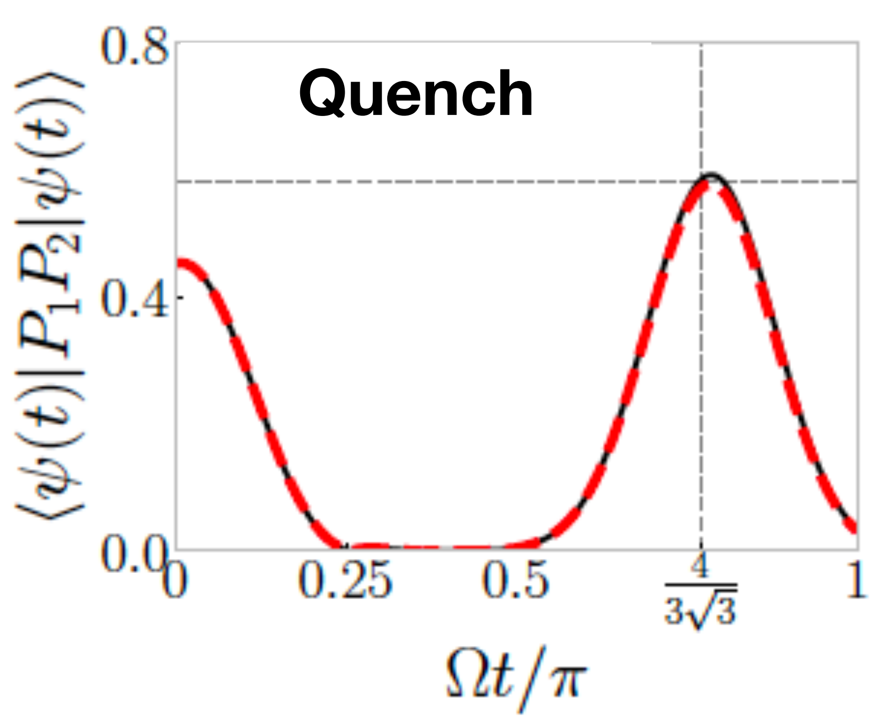
Measuring Off-Diagonal String Operators

→ Diagonal string P measured from snapshots of atoms

→ Off-diagonal string Q can be reduced to P after rotation

e^{iHt} () $e^{-iHt} =$  for $\Omega t = \frac{4\pi}{3\sqrt{3}}$

using $H = \frac{\Omega}{2} \sum_i P \sigma_i^y P$ where P only enforces blockade inside triangles!

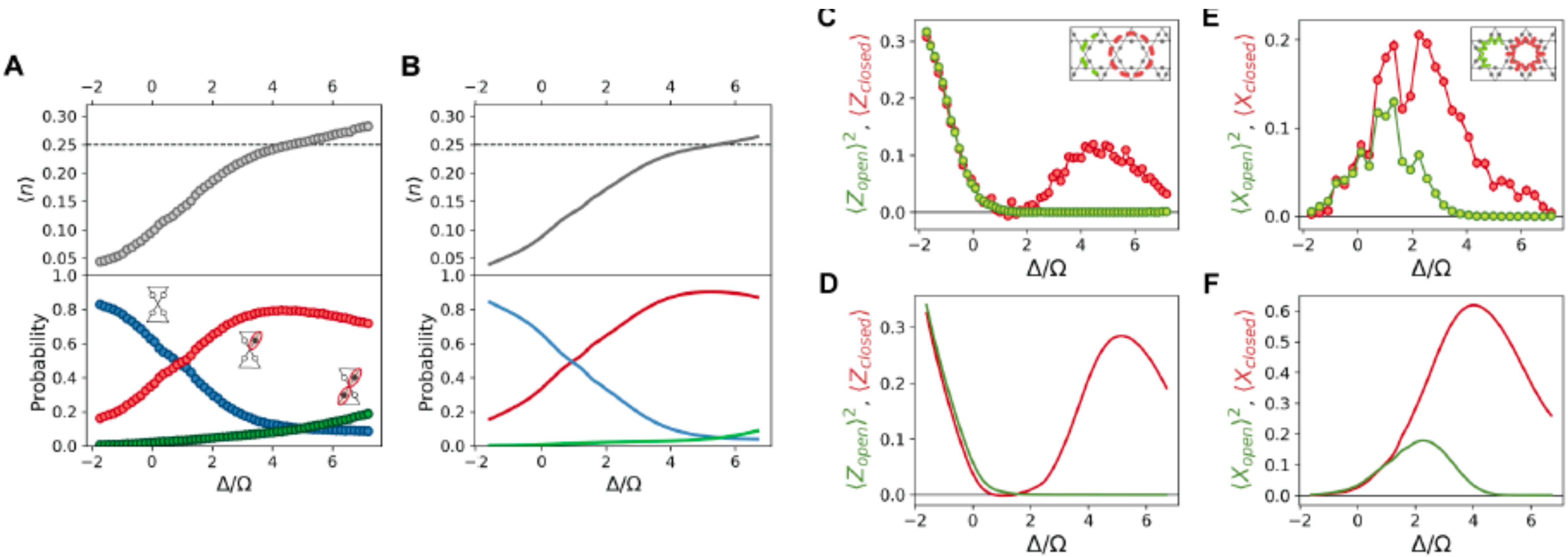


Numerical Simulation of Sweep Dynamics Versus Experiment

Semeghini et al. Science `22. arxiv:2104.04119

Supplementary

Diagonal Off-Diagonal



EXPERIMENT vs **NUMERICS** (**Dynamics**)

Open Strings are systematically smaller than **closed ones** of the same size in the **`Spin Liquid'**

arXiv:2011.12310 [pdf, other]

Prediction of Toric Code Topological Order from Rydberg Blockade

Ruben Verresen, Mikhail D. Lukin, Ashvin Vishwanath

Comments: v2: updates include a confirmation that the spin liquid on a ruby lattice (for choice of lattice parameter rho=3) persists upon including long-range Van der Waals interactions. v3: final published version

Journal-ref: Phys. Rev. X 11, 031005 (2021)

Subjects: **Strongly Correlated Electrons (cond-mat.str-el)**; Quantum Gases (cond-mat.quant-gas); Atomic Physics (physics.atom-ph); Quantum Physics (quant-ph)

Theory+ Numerics+Experiment Collaboration

Mikhail D. Lukin

Ruben Verresen

arXiv:2104.04119 [pdf, other]

Probing Topological Spin Liquids on a Programmable Quantum Simulator

Giulia Semeghini, Harry Levine, Alexander Keesling, Sepehr Ebadi, Tout T. Wang, Dolev Bluvstein, Ruben Verresen, Hannes Pichler, Marcin Kalinowski, Rhine Samajdar, Ahmed Omran, Subir Sachdev, Ashvin Vishwanath, Markus Greiner, Vladan Vuletic, Mikhail D. Lukin

Subjects: **Quantum Physics (quant-ph)**; Quantum Gases (cond-mat.quant-gas); Atomic Physics (physics.atom-ph)

Giulia Semeghini | Harry Levine | Alexander Keesling | Sepehr Ebadi | Tout T. Wang | Dolev Bluvstein | Hannes Pichler | Marcin Kalinowski | Rhine Samajdar | Ahmed Omran | Subir Sachdev | Markus Greiner | Vladan Vuletic